Weak convergence methods in Calculus of Variations and PDEs - SS 2009

1. Weak convergence and compactness, review of basic theory ([3], [4])
    1.1 Lebesgue and Sobolev spaces
    1.2 Radon measures, reduced defect measures, a refinement of Fatou's Lemma
2. Concentrated compactness ([4], [5, 6])
    2.1 The critical case $W^{1,p} \hookrightarrow L^{p*}$, 2^{nd} Concentration-compactness Lemma
    2.2 Minimizers for critical Sobolev nonlinearities
    2.3 Concentration-cancellation, an application to Euler equations
3. Compensated compactness ([4], [8])
    3.1 Continuity for $f(u_j)$ w.r.t. weak$*$ topology, Young measures
    3.2 Generalized *div-curl* Lemma
    3.3 An application to scalar conservation laws
4. Γ-convergence ([2] [1])
    4.1 Definition, main properties, and examples on $\mathbb{R}$
    4.2 The Modica Mortola functional
5. Quasiconvexity ([4], [7])
    5.1 A Variational model for microstructures, rank-1 connection, quasiconvexity
    5.2 Gradient Young measures, semicontinuity and relaxation of energies

References

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