

① Data $\sum \frac{3^{2n}}{(2n)!} \cdot x^{2n}$

(a) Criterio del rapporto:

$$\frac{a_{n+1}}{a_n} = \frac{3^{2(n+1)}}{(2(n+1))!} \cdot \frac{(2n)!}{3^{2n}} = \frac{3^2}{(2n+2)(2n+1)} \longrightarrow 0 \text{ per } n \rightarrow +\infty$$

$\Rightarrow$ Insieme di convergenza: $\mathbb{R}$

(b) e (c): vedi pag. 3

② $f(x,y) = e^{x^2/2} \cdot \cos(y)$

Punti stazionari

$$\partial_x f = x e^{x^2/2} \cos(y) \Rightarrow \nabla f = 0 \text{ in } (0, k\pi), k \in \mathbb{Z}$$

$$\partial_y f = -e^{x^2/2} \sin(y)$$

$$\partial_{xx}^2 f = (1+x^2) e^{x^2/2} \cos y$$

$$\partial_{xy}^2 f = -x e^{x^2/2} \sin(y)$$

$$\partial_{yy}^2 f = -e^{x^2/2} \cos(y)$$

La matrice Hessiana nei p.ti stazionari è quindi

$$HF = \begin{pmatrix} \cos(k\pi) & 0 \\ 0 & -\cos(k\pi) \end{pmatrix}$$

$\det(HF) < 0 \Rightarrow$ Tutti i p.ti $(0, k\pi)$ sono di sella
(o colle)

③ Parametrizzo il segmento AB con $\gamma(t) = (t, t)$, $t \in [1, 2]$

$$\gamma'(t) = (1, 1)$$

$$L = \int_{\gamma} F \cdot d\gamma = \int_1^2 F(\gamma(t)) \cdot \gamma'(t) dt = 2 \int_1^2 2t (1 + \log t^2) dt$$

per parti

$$= -2 \int_1^2 t^2 \cdot \frac{2}{t} dt + 2 \left[t^2 (1 + \log t^2) \right]_{t=1}^{t=2}$$

$$= -2 [t^2]_1^2 + 2 [t^2]_1^2 + 8 \log 4 - 2 \log 1 = 8 \log 4$$

④ $g(x, y) = xy$, Σ è il grafico di g .

$$\Rightarrow dS = \sqrt{1 + |vg|^2} dx dy = \sqrt{1 + x^2 + y^2} dx dy$$

$$\iint_{\Sigma} \frac{x + 2y + x^3}{x} dS = \iint_D (1 + x^2 + y^2) \sqrt{1 + x^2 + y^2} dx dy$$

(coord. polari)

$$= \int_{\frac{\pi}{2}}^{\frac{3}{2}\pi} \int_1^2 (1 + \rho^2)^{\frac{3}{2}} \rho d\rho d\theta$$

$$u = 1 + \rho^2 \quad du = 2\rho d\rho \quad (\text{sostituzione})$$

$$\dots = \pi \int_2^5 \frac{u^{\frac{3}{2}}}{2} du = \frac{\pi}{2} \left[\frac{2}{5} u^{\frac{5}{2}} \right]_2^5$$

$$= \frac{\pi}{5} \left[5^{\frac{5}{2}} - 2^{\frac{5}{2}} \right]$$

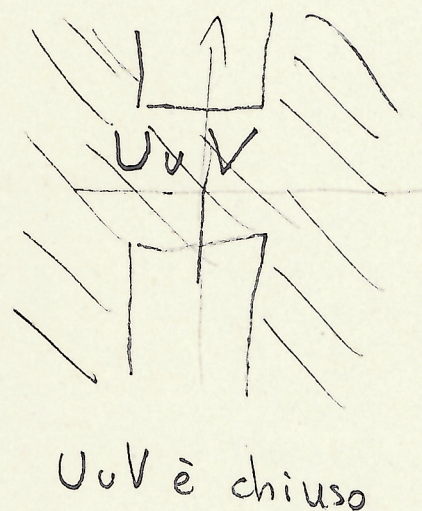
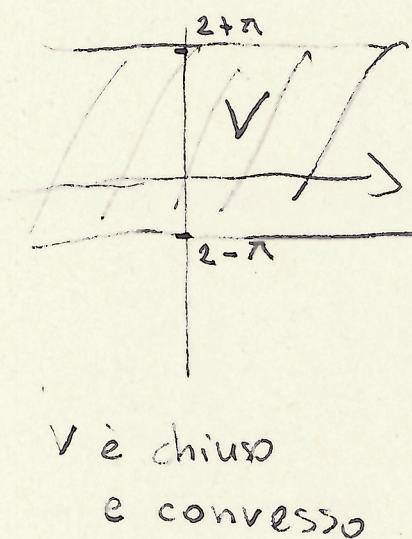
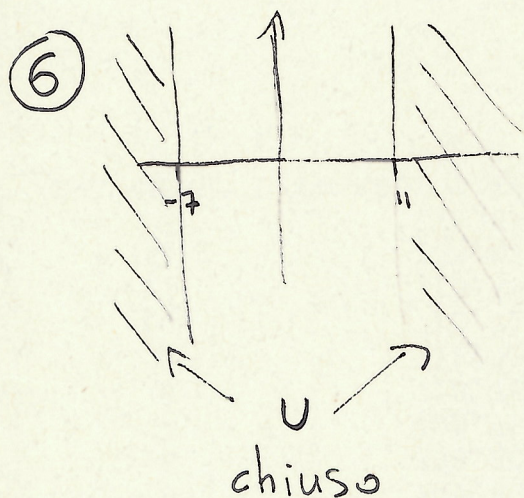
① (b) se $x > 0$ $x^{5n} = (x^{5/2})^{2n}$

$$\sum_{n=0}^{+\infty} \frac{3^{2n} x^{5n}}{(2n)!} = \sum_{n=0}^{+\infty} \frac{(3x^{5/2})^{2n}}{(2n)!} = \cosh(3x^{5/2})$$

(c) se $x < 0$ $x^{5n} = (-x)^{5n} \cdot (-1)^n = ((-x)^{5/2})^{2n} \cdot (-1)^n$

$$\sum_{n=0}^{+\infty} \frac{3^{2n} x^{5n}}{(2n)!} = \sum_{n=0}^{+\infty} \frac{(-1)^n (3 \cdot (-x)^{5/2})^{2n}}{(2n)!} = \cos(3 \cdot (-x)^{5/2})$$

⑤ $\iint_{\partial E} F \cdot n \, dS = \iiint_E \operatorname{div} F \, dx dy dz = 2 \operatorname{Vol}(E) = 36\pi$



⑨ $F(x, y, z) = x + y^2 + z^3 - 4$ $P_0 = (2, 1, 1)$ Piano tangente:
 $0 = \nabla F(x_0, y_0, z_0) \cdot (x - x_0, y - y_0, z - z_0) = (1, 2, 3) \cdot (x - 2, y - 1, z - 1)$
 $\Rightarrow 1x + 2y + 3z = 7$

L'unico p.to che non verifica l'equazione è $(4, 1, 0)$

$$(10) \quad \iint_{\partial D} \operatorname{rot}(\nabla f) \cdot n \, dS = 0 \quad (\operatorname{rot}(\nabla f) = 0)$$

|| $\leftarrow$ Stokes

$$\int_{\partial D} \nabla f \cdot dr$$

$$\iiint_{\Sigma} \operatorname{div}(\operatorname{rot} F) \, dx dy dz = 0 \quad (\operatorname{div}(\operatorname{rot} F) = 0)$$

|| $\leftarrow$ Gauss

$$\iint_{\partial \Sigma} \operatorname{rot} F \cdot n \, dS$$

$$\Rightarrow \text{L'unico } \neq 0 \text{ } \bar{e} \quad \iiint_{\Sigma} \operatorname{div}(\nabla f) \, dx dy dz.$$

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