

- I want to take the BASIC EXAM
 I want to take the ADVANCED EXAM

Exam rules:

- Basic exam: the maximum grade is 24/30.
- Advanced exam: the maximum grade is 30/30 cum laude.

Total time is 1 hour. Students who get a positive grade in the written part (i.e., at least 18/30) *might* choose to take an oral exam. For students who choose the basic written exam, the maximum grade obtainable can never exceed 24/30.

BASIC EXAM

1. Given the function $f(x) = \cos(2\pi x)$ compute its Lagrange interpolant of degree 2 through the points $x_1 = 0$, $x_2 = 1/2$, $x_3 = 1$.
2. Write the pseudocode of the Gaussian elimination method, without pivoting, and apply it to solve the linear system $Ax = b$, where

$$\begin{bmatrix} 2 & 3 & 1 \\ 1 & 1 & 2 \\ 1 & 3 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

showing the intermediate computations.

ADVANCED EXAM

3. Write the pseudocode of the power method for the computation of the dominant eigenvalue and eigenvector. State conditions that guarantee its convergence. Starting from $v^{(0)} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, compute 2 iterations of the power method on the matrix

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

showing the intermediate steps.

4. Introduce splitting methods to solve linear systems, and their pseudocode (possibly including stopping criteria). Then, give a sufficient condition on the iteration matrix that implies the convergence of the splitting method. Finally, describe the particular case of the Jacobi method, state and prove its convergence under the assumption that the matrix of the system is diagonally dominant.