

Course Title. *Real Geometry*

Teacher(s). Ludovico Pernazza

Overview. The course is devoted to introduce the main themes and tools of real algebraic and analytic geometry: Hilbert's 17th problem, Artin–Lang theory, semialgebraic and semianalytic sets, real algebra, relations between the algebraic and analytic frameworks. The prerequisites are a good knowledge of basic topology and algebra (mainly ring theory).

When. Spring 2023

Where. Pavia, Dipartimento di Matematica

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Abstract. Real geometry: real algebraic varieties, semialgebraic sets.

Real algebra: ordered fields, real ideals, real radical of an ideal, real closure of a field.

Hilbert's 17th problem on sums of squares.

Real spectrum, fans, order spaces; real Nullstellensatz and Positivstellensatz.

Basic and principal semialgebraic sets, separation of semialgebraic sets and Bröcker's theorem.

Real analytic varieties, extension of orderings and Artin's approximation theorem.

Nash functions, Pfaffian functions, o-minimal structures and other real rings of functions.

Global real analytic geometry.

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C. Andradas, J.M. Ruiz, Algebraic and analytic geometry of fans, Memoirs of the AMS, 0553 (115), American Mathematical Society, Providence 1995

J. Bochnak, M. Coste, M.-F. Roy, Real Algebraic Geometry, Ergebnisse der Mathematik und ihrer Grenzgebiete (3), vol. 36, Springer-Verlag, Berlin 1998

M.A. Marshall, Spaces of Orderings and Abstract Real Spectra, Lecture Notes in Mathematics 1636, Springer, Berlin Heidelberg New York, 1996

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