Traveling Salesman Problem (TSP)

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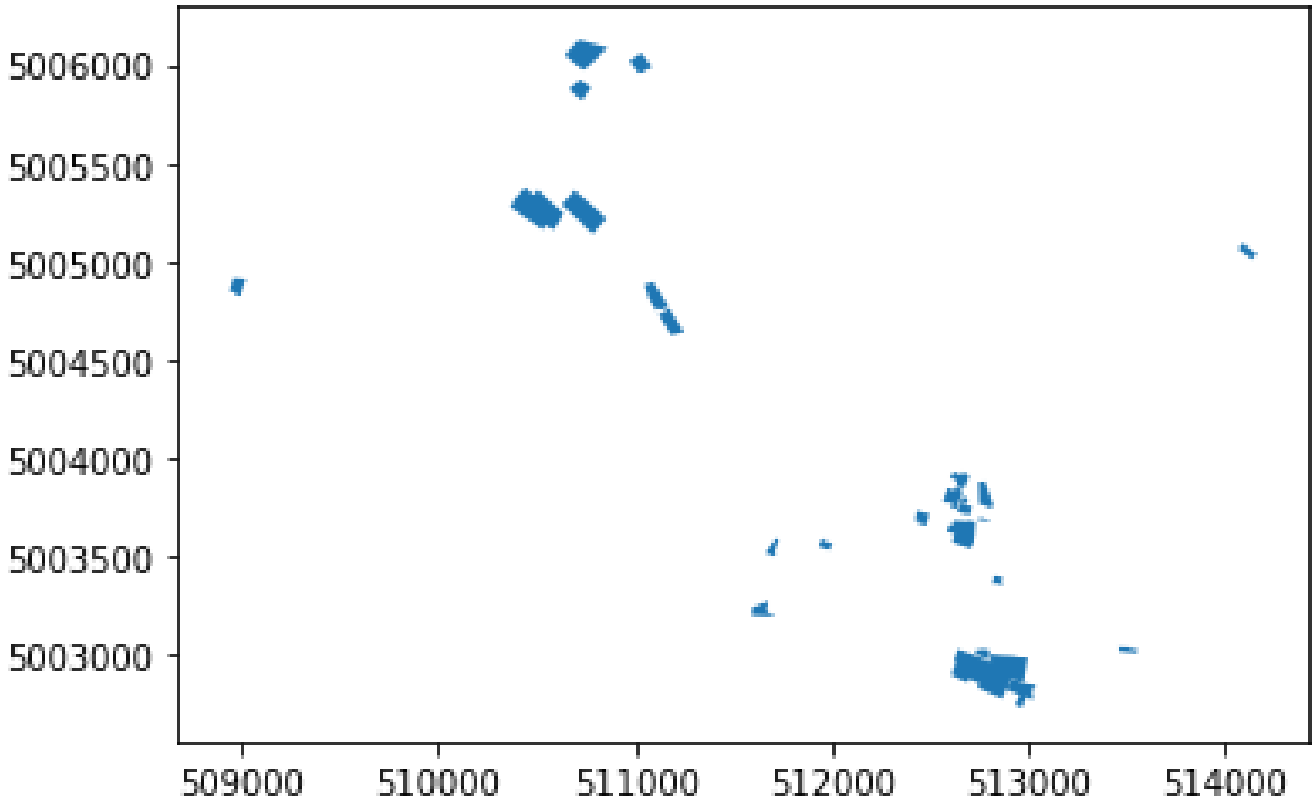
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blog: http://stegua.github.com

Traveling Student Problem (TSP): A student from the University of Pavia must sell the tickets for the next post-Covid19 re-opening party, planned on May, 1st, 2021. For this reason, she must visit all the n residences of the city exactly once, and then she has to return back to her own residence. The time taken to go from residence i to residence j is c_{ij}, and the visiting time is fixed for each residence.

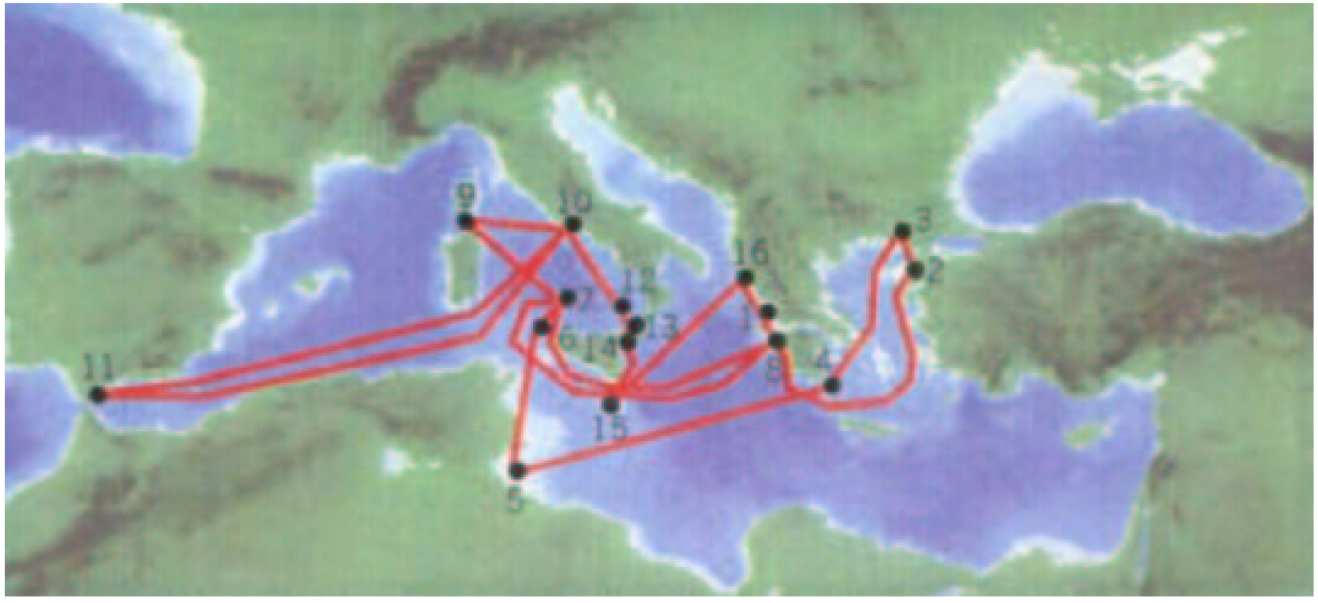
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LAB SESSION: How can you formulate this problem using ILP?

University Residences - Open Data Comune di Pavia



EDITORIAL



Martin Grötschel and Manfred Padberg,

Figure 1 – The legendary Ulysses was almost hopelessly lost on the two-dimensional sea surface and he didn't choose the shortest way home (picture, after the interpretation of E. Bradley). To not just get back home, but to find the shortest roundtrip through all 16 spots, you have found your way around in a 120-dimensional space.

https://www.zib.de/groetschel/pubnew/paper/groetschelpadberg2001a.pdf


HELP! WE'RE LOST!
HELP "CAR 54"...AND WIN CASH
54...$1,000 PRIZES
ONE...$10,000 GRAND PRIZE
START and FINISH
Lewiston
Portland
Butte
Boise
Twin Falls
Reno
Salt Lake City
Colorado Springs
Mexican Hat
Lone Pine
Marble Canyon
Manuelito
Truth or Consequences
Amarillo
Wichita
Kansas City
Lincoln
Chicago
La Crosse
Marion
Indianapolis
Erie
Carlisle
Wana
Chattanooga
Little Rock
Baton Rouge
Bainbridge
Map by Rand McNally
Help Toody and Muldoon find the shortest round trip route to visit all 33 locations shown on the map.
All you do is draw connecting straight lines from location to location to show the shortest round trip route.
HERE'S THE CORRECT START...
Begin at Chicago, Illinois. From there, lines show correct route as far as Erie, Pennsylvania. Next, do you go to Carlisle, Pennsylvania or Wana, West Virginia? Check the easy instructions on back of this entry blank for details.
OFFICIAL RULES ON REVERSE SIDE
©PROCTER & GAMBLE 1962

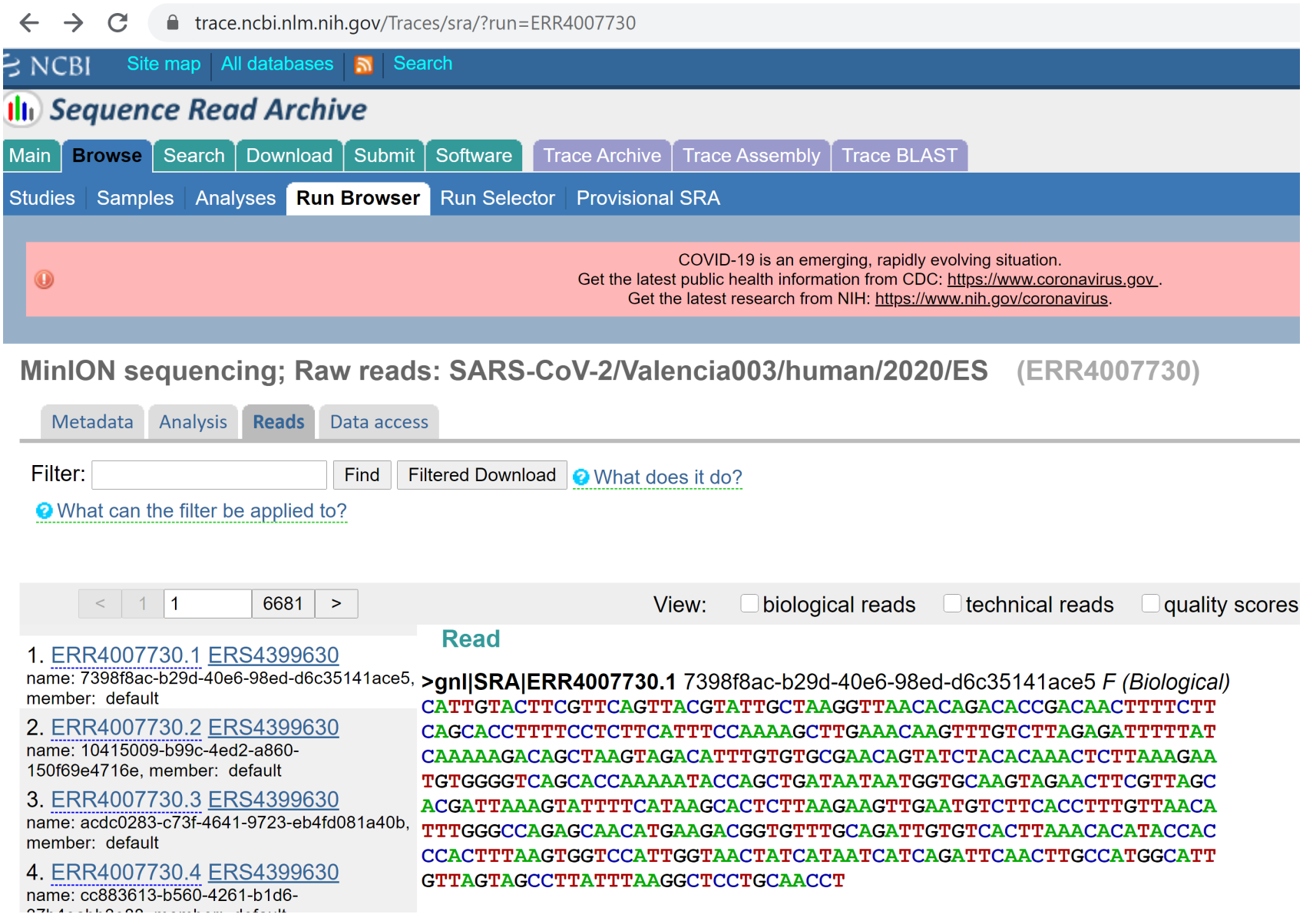
SCIENCE



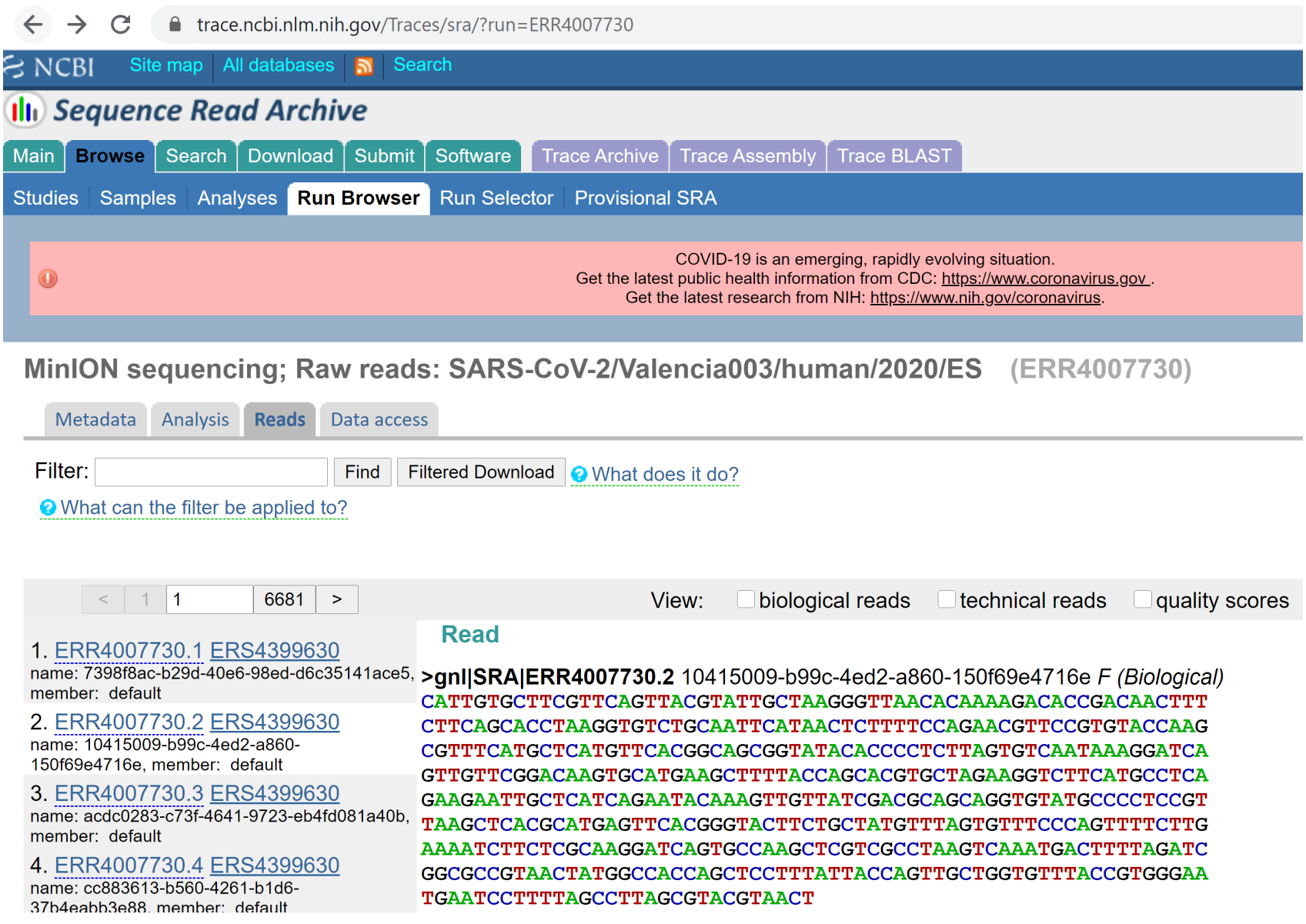
FINDING the shortest route for a traveling salesman—starting from a given city, visiting each of a series of other cities, and then returning to his original point of departure—is more than an after-dinner teaser. For years it has baffled not only goods- and salesmen-routing businessmen but mathematicians as well. If a drummer visits 50 cities, for example, he has 10^{62} (62 zeros) possible itineraries. No electronic computer in existence could sort out such a large number of routes and find the shortest.

Three Rand Corp. mathematicians, using Rand McNally road-map distances between the District of Columbia and major cities in each of the 48 states, have finally produced a solution (see above). By an ingenious application of linear programming—a mathematical tool recently used to solve production-scheduling problems—it took only a few weeks for the California experts to calculate "by hand" the shortest route to cover the 49 cities: 12,345 miles.

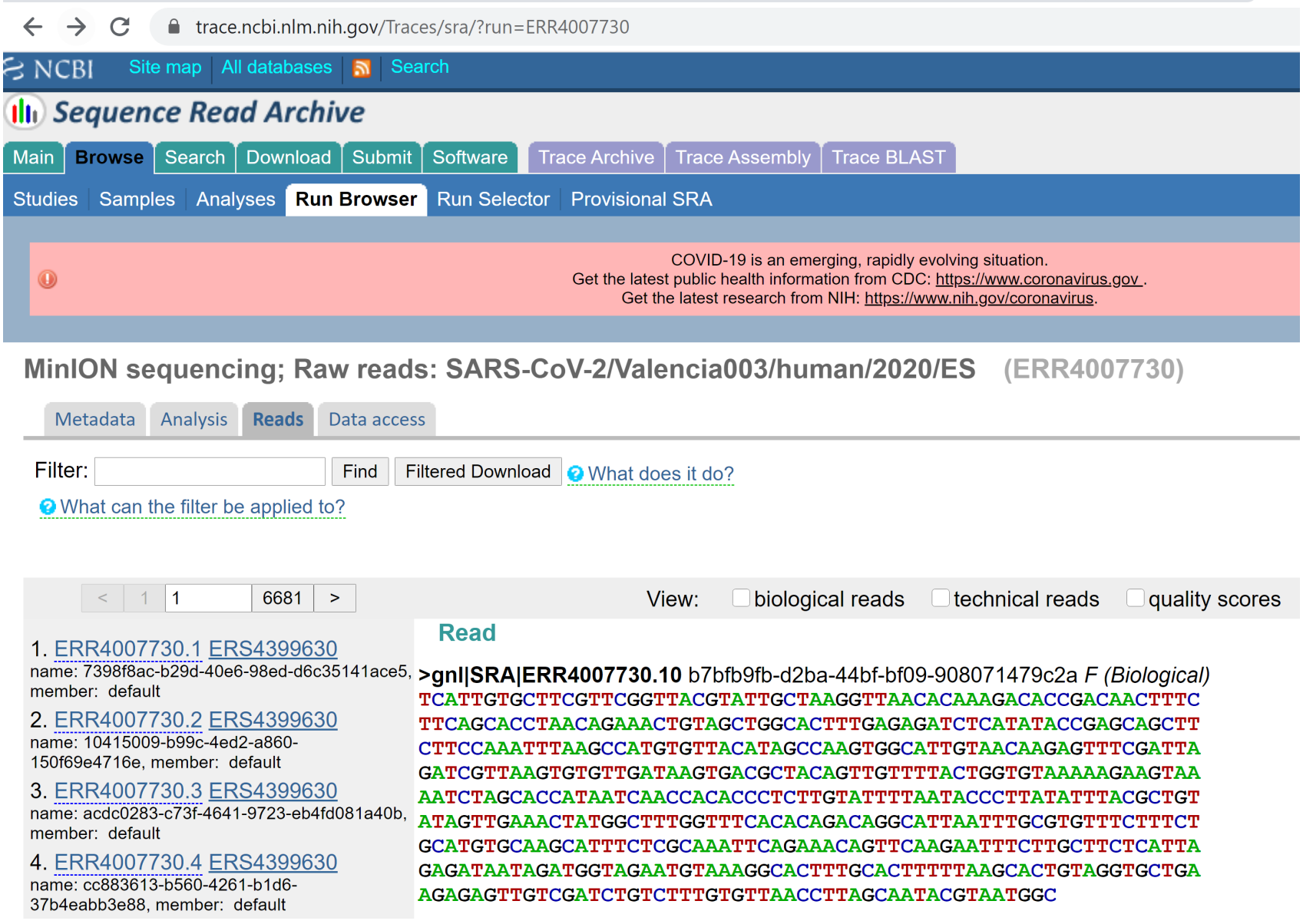
TSP: The DNA Sequencing Problem 1/1



TSP: The DNA Sequencing Problem 1/1



TSP: The DNA Sequencing Problem 1/1



TSP: The DNA Sequencing Problem 2/2

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TSP: The DNA Sequencing Problem 2/2

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TSP: The DNA Sequencing Problem 2/2

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TSP: The DNA Sequencing Problem 2/2

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TSP: The DNA Sequencing Problem 2/2

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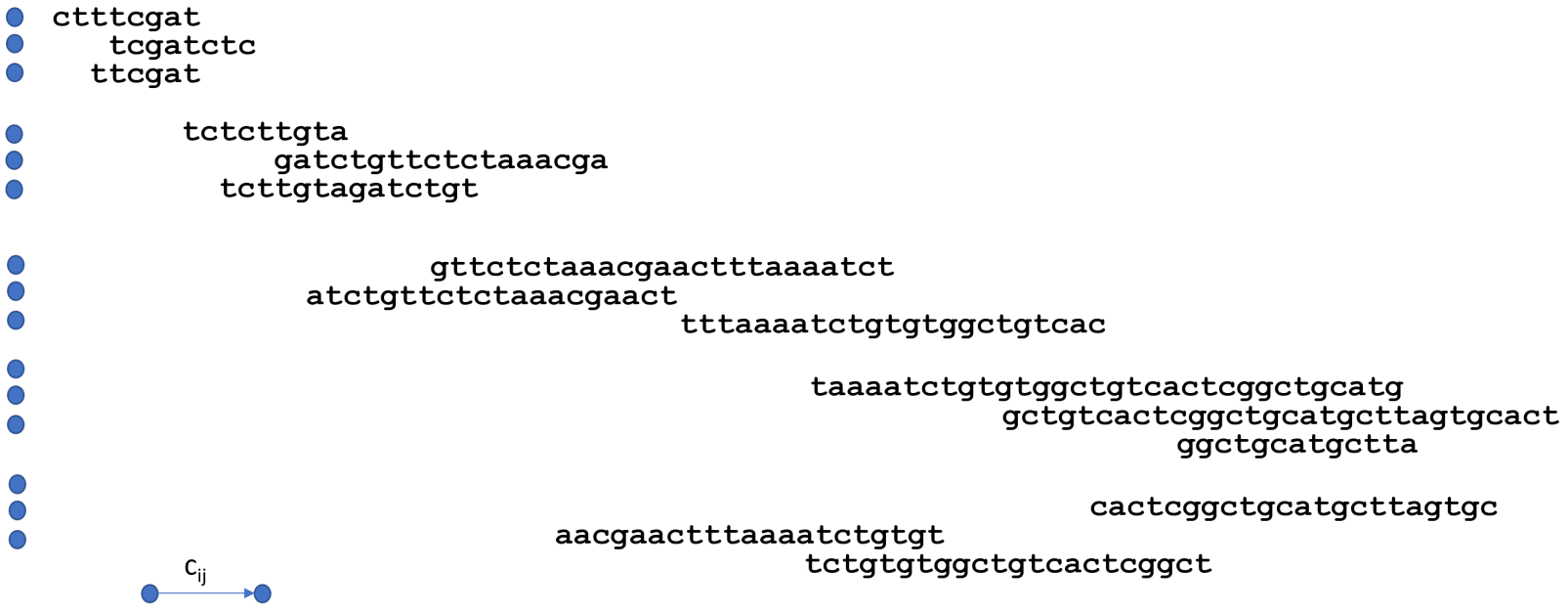
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TSP: The DNA Sequencing Problem 2/2

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TSP: The DNA Sequencing Problem 2/2



Solve a TSP on the directed graph defined with a node for each DNA fragment (*read*), with cost c_{ij} given by the probability that fragment i follows/overlap with fragment j. Later, reconstruct the overall sequence in post-processing.

https://www.ncbi.nlm.nih.gov/nuccore/MT509704

TSP: Other Genomics Applications

1. **The Marker-Ordering Problem**
   (consecutive 1 block matrices as subproblem)
2. **The Signaling Pathways in Cervical Cancer**

Project 7c on the list of papers, Chap. 9 of ILP for Computational and System Biology.

References for these slides

- The Optimized Odyssey, by Martin Grotschel and Manfred Padberg. https://www.zib.de/groetschel/pubnew/paper/groetschelpadberg2001a.pdf
- Chap. 9 in Integer Linear Programming in Computational and Systems Biology. Dan Gusfield. Cambridge University Press, 2019