Phase transition with memory

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As is well known, the energy balance for the internal energy and the heat flux yields the standard heat equation, provided that the constitutive law for the heat flux $\mathbf{q}$ is the Fourier law, namely

$$\mathbf{q}(x,t) = \mathbf{q}_0(x) - k_0 \nabla\vartheta(x,t) \tag{1}$$

where ϑ is the temperature and $\mathbf{q}_0$ is known. Let us replace (1) with the law

$$\mathbf{q}(x,t) = \mathbf{q}_0(x) - k_0 \nabla\vartheta(x,t) - \int_0^t k(t-s)\nabla\vartheta(x,s)\, ds \tag{2}$$

which accounts also on the past history of the temperature through the convolution term. Equation (2) is the Coleman–Gurtin or the Gurtin–Pipkin law according to whether k_0 is positive or zero, and leads to an equation whose principal part is parabolic or hyperbolic, respectively, at least when k is a decreasing exponential.

A similar situation happens for phase transition models, like the Stefan problem or the phase field system, when (1) is replaced by (2). In this talk, a brief overview on the results obtained in this field is given, with more detail on some selected problems.