Conjectures

- Homological mirror symmetry.

(1) $[D_i \cdot D_j]$ is NOT ≤ 0.
(i.e. the "positive" case, for which U is affine).

$B + i\omega$:= a fixed complexified Kähler form on Y.

Project $[B + i\omega]$ to $H^2(Y, \mathbb{C})/H^2(Y, \mathbb{Z})$.
then, we can think of

$$\delta := \exp(2\pi i \, [B + i\omega])$$

as a point of
$$T_Y = \mathrm{Pic}(Y) \otimes \mathbb{C}^*.$$

The GHK family is defined globally

$$\mathcal{X} \longrightarrow \operatorname{Spec} \mathbb{C}[NE(Y)] .$$
$$\cup$$
$$T_Y$$

Conjecture. (i) Suppose the fibre $\mathcal{X}_s$ is smooth. then,

[illegible] $) \cong D^b(\mathcal{X}_s)$

$D''F_{wr}(U, (B+i\omega)|_U) = D(\mathfrak{X}_s)$.

(ii) In general, $\mathfrak{X}_s$ has quotient singularities $\mathbb{C}^2/G$, $G \subset SL(2,\mathbb{C})$ finite.
Then, the same statement holds provided we regard $\mathfrak{X}_s$ as an orbifold, or if we replace $\mathfrak{X}_s$ by its minimal resolution.

__Rmk.__ Recall that GHK prove (announce a proof) that in this case

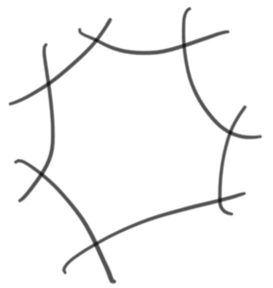
$$\mathfrak{X} \longrightarrow T_Y$$

is a ("canonical") family of versal deformations of (U, Ω).

(Discuss applications to symplectic cohomology).

(2) $[D_i \cdot D_j] \leq 0$ but not < 0.

$\Rightarrow$ after possibly contracting all -1 curves,

$D =$  ; $D =$  .

(−2 curves)

Generically in moduli, $\exists$ elliptic fibration

$$w : Y \longrightarrow \mathbb{P}^1$$

with $D = \pi^{-1}(\infty)$.

In this case, the mirror is

$$\mathcal{X} \longrightarrow \{|z^D| < 1\} \subset \operatorname{Spec} \mathbb{C}[P].$$

Fix

$$s = \exp(2\pi i[B + i\omega]) \in \{|z^D| < 1\}.$$

Then, $\mathcal{X}_s$ is (in a canonical way) the complement of an anticanonical smooth (elliptic) curve in a del Pezzo surface Z_s (of deg k).

Conjecture.

(i) $$\underbrace{\mathbb{F}S(U, B+i\omega, w)}_{\text{"Fukaya-Seidel category of vanishing cycles"}} \cong D^b(Z_s).$$

(ii) $$D^\pi \mathbb{F}_{wr}(U, B+i\omega, w) \cong D^b(\mathcal{X}_s).$$

• **Periods.** Suppose $[D_i \cdot D_j]$ is NOT < 0.

So we have the GHK family

$$\mathfrak{X} \longrightarrow \underbrace{S' \subset S}_{\substack{\text{analytic open subset} \\ \text{(possibly all } \operatorname{Spec}\mathbb{C}[P]!)}} := \operatorname{Spec}\mathbb{C}[P]$$

as above.

Set $S'^{o} :=$ locus of smooth fibres.

$\forall s \in S'^{o}$, there is a canonical identification

$$H_2(\mathfrak{X}_s, \mathbb{Z}) / \langle \gamma_s \rangle = H_2(U, \mathbb{Z}) / \langle \gamma \rangle$$
$$= \langle D_1, \dots, D_n \rangle^{\perp}$$
$$=: Q,$$

where γ_s, γ are the classes of certain tori.

Pick normalised holo. sympl. form so

$$\int_{\gamma_s} \Omega = 1.$$

Conjecture.

(i) The local system

$$S'^{o} \ni s \longmapsto H_2(\mathfrak{X}_s, \mathbb{Z}) / \langle \gamma_s \rangle$$

is trivial (ie it has trivial monodromy).

(iii) $\forall \beta \in Q = H_2(X_s, \mathbb{Z})/\langle \gamma_s \rangle$, $\forall$ lift $\tilde{\beta} \in H_2(X_s, \mathbb{Z})$, we have $z^\beta = \exp\left(2\pi i \int_{\tilde{\beta}} \Omega\right)$.