GENERIC-based thermodynamically consistent time integration methods for coupled dissipative problems

Sergio Conde Martín and Juan Carlos García Orden

ETSICCP, Dpto. Mecánica de Medios Continuos y Teoría de Estructuras, Technical University of Madrid, Spain

sergio.conde@upm.es

November 25, 2014

ABSTRACT

The concept of thermodynamically consistent (TC) time integration schemes encompasses any numerical integration method intended to solve thermodynamical systems in such a way that the laws of thermodynamics are discretely satisfied by design. In the particular case of reversibility, i.e. Hamiltonian systems, a well-known example is the energy-momentum method due to Simó and Tarnow [1]. Recently, Romero [3, 2] introduced a general framework to attain thermodynamically consistent time integration methods for evolution systems with irreversible processes such as those with internal dissipation mechanisms (viscoelasticity, plasticity, damage). This generalization was made possible by the use of GENERIC form [4] of the evolution equations of the thermodynamical system at hand.

The election of the thermodynamical variable was reported to become crucial in the formulation. Thus, the GENERIC-based TC integrators newly proposed have so far been based on the entropy as thermodynamical state variable [2, 5, 6], since it was reported to best fit in the GENERIC formalism. However, this choice obliged to assume important restrictions to the formulation, such as the necessity for material models to enable the analytical provision of its potentials in terms of the entropy and, more importantly, the incapability to impose Dirichlet's boundary conditions. These reasons motivated the search for the formulation of GENERIC-based TC integrator in terms of temperature which we have successfully tackled overcoming the above limitations.

Within this new framework, we present a brief review of TC integration methods for nonlinear isothermal viscoelastodynamics [7], discrete thermoelastodynamics [8] and thermo(visco)elastodynamics using temperature as state variable. All these methods exactly preserve the continuum laws of thermodynamics intrinsically while preserving the existing symmetries. The resulting solutions are physically accurate since they preserve the fundamental physical properties of the model. What is more, they show excellent performances with respect to methods' robustness and stability. Proof for these claims will be provided in the presentation as well as numerical examples that illustrate their performance.

References

[1] J.C. Simó, N. Tarnow. The discrete energy-momentum method. Conserving algorithm for nonlinear elastodynamics. *Zeitschriftfür Angewandte Mathematik und Physik (ZAMP)*, Vol. **43**(5), 757–792, 1992.

[2] I. Romero. Thermodynamically consistent time-stepping algorithms for nonlinear thermomechanical systems. *International Journal for Numerical Methods in Engineering*, 79(March):706–732, 2009.

[3] Ignacio Romero. Algorithms for coupled problems that preserve symmetries and the laws of thermodynamics Part I: Monolithic integrators and their application to finite strain thermoelasticity. *Computer Methods in Applied Mechanics and Engineering*, Vol. **199**, 1841–1858, 2010.

[4] Öttinger. *Beyond Equilibrium Thermodynamics.* John Wiley & Sons, New Jersey, 2005.

[5] J. C. García Orden and I. Romero. Energy-Entropy-Momentum integration of discrete thermo-visco-elastic dynamics. *European Journal of Mechanics - A/Solids*, 32:76–87, October 2011.

[6] M. Krüger, M. Groß, and P. Betsch. A comparison of structure-preserving integrators for discrete thermoelastic systems. *Computational Mechanics*, 47(6):701–722, January 2011.

[7] S. Conde Martín, J. C. García Orden, and I. Romero. Energy-consistent time integration for nonlinear viscoelasticity. *Computational Mechanics*, 54(2):473–488, 2014.

[8] S. Conde Martín, P. Betsch, and J. C. García Orden. A temperature-based thermodynamically consistent integration scheme for discrete thermo-elastodynamics. Submitted in September 2014.