Towards relaxation of variational problems in nonlinear elasticity

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This talk is motivated by the relaxation of minimization problems for the functional $J(y) := \int_\Omega W(\nabla y(x))\,\mathrm{d}x$, where $W(F)$ tends to infinity if the determinant of F converges to zero. and $W(F) = +\infty$ if $\det F \le 0$. In order to capture this behavior we make W to depend not only of the deformation gradient but also on its inverse, for instance, or on other quantities from the so-called Seth-Hill family of strain measures. Having a uniform bound on $\{\nabla y_k\}$ as well as on $\{(\nabla y_k)^{-1}\}$ in $L^p(\Omega;\mathbb{R}^n)$ we are led to study Young measures generated by matrix-valued mappings $\{Y_k\}_{k\in\mathbb{N}} \subset L^p(\Omega;\mathbb{R}^{n\times n})$, such that $\{Y_k^{-1}\}_{k\in\mathbb{N}} \subset L^p(\Omega;\mathbb{R}^{n\times n})$ is bounded, too. Moreover, the constraint $\det Y_k > 0$ can be easily included and is reflected in a condition on the support of the measure. Finally, we discuss the case $Y_k := \nabla y_k$ and state necessary conditions for the relaxation. This is an ongoing work with Barbora Benešová and Gabriel Pathó (Prague).