Pressure and phase transition in Potts models in Statistical Mechanics

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Abstract

In this talk we discuss the notion of pressure P, (the grand canonical energy or potential), in Potts models with n different kinds of atoms. P is a convex function on $\mathbb{R}^n$. We give a mathematical explanation why there is a first order phase transition in the places where P is not differentiable. We also discuss the relative entropy of the Potts model, as a function of densities of the atoms of each kind. This function is the minus of the dual, (Legendre), convex function corresponding to P. We point out a reliable fast algorithm to compute P. The application of this algorithm to the computation of the first and the second derivatives of P may lead to new methods of finding numerically the points of phase transitions in Potts models in general, and in Ising model in particular. This is work in progress with Uri N. Peled.

References

[1] S. Friedland and U.N. Peled, Theory of Computation of Multi-dimensional Entropy with an Application to the Monomer-Dimer Problem, *Advances of Applied Math.* 34(2005), 486-522.

[2] S. Friedland, Multi-dimensional entropy and the monomer-dimer problem, Slides of lectures in HU, Technion, TAU, December, 2003,

http://www2.math.uic.edu/ friedlan/colhu.pdf