Scales in mathematical fluid dynamics

Eduard Feireisl

Many interesting problems in mathematical fluid mechanics involve the behavior of solutions to systems of nonlinear partial differential equations as certain parameters vanish or become infinite. Frequently the solutions converge, provided the limit exists, to a solution of a limit problem represented by a qualitatively different system of differential equations. The simplest physically relevant example of this phenomenon is the behavior of a compressible fluid flow in the situation when the Mach number tends to zero, where the limit solution formally satisfies a system describing the motion of an incompressible fluid. Other interesting phenomena occur in the equations of magnetohydrodynamics, when either the Mach or the Alfven number, or both, tend to zero. As a matter of fact, most, if not all mathematical models used in fluid mechanics rely on formal asymptotic analysis of more complex systems. The concept of incompressible fluid itself should be viewed as a convenient idealization of a medium in which the speed of sound dominates the characteristic velocity.

In the lecture, we discuss several approaches to scaling and the related problems of singular limits arising in the analysis of complete fluid systems. In particular, we focus on the methods based on the concept of weak and dissipative solutions. We review some recent results concerning existence of solutions and their singular limits. In such a way, some well known systems of partial differential equations are identified as certain limits of complete fluids.