Parabolic equations with p, q-growth

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In this talk we will present existence and regularity results for parabolic problems with p, q-growth. The results cover for example equations and systems of the type

$$\partial_t u - \sum_{i=1}^{n} \frac{\partial}{\partial x_i} \Big(\big(\mu^2 + |D_i u|^2\big)^{\frac{p_i - 2}{2}} D_i u \Big) = 0$$

with $\mu \in [0, 1]$ and suitable growth exponents p_i.

We will start considering the case of equations and prove that any weak solution $u \in L^p(0, T; W^{1,p}(\Omega)) \cap L^q_{\mathrm{loc}}(0, T; W^{1,q}_{\mathrm{loc}}(\Omega))$ admits a locally bounded spatial gradient Du, provided that

$$2 \le p \le q < p + \frac{4}{n}.$$

Moreover, the stronger assumption

$$2 \le p \le q < p + \frac{4}{n+2}$$

guarantees an existence result for the associated Cauchy-Dirichlet problem.

In the second part of the talk we turn our attention to the vectorial case and consider the evolution problem associated with a convex integrand $f \colon \mathbb{R}^{Nn} \to [0, \infty)$ satisfying a non-standard p, q-growth assumption. To establish the existence of solutions we introduce the concept of *variational solutions*. In contrast to weak solutions, i.e. mappings $u \colon \Omega_T \to \mathbb{R}^n$ which weakly solve Euler equation, variational solutions exist under much weaker assumptions on the gap $q - p$. Here, we prove the existence of variational solutions provided the integrand f is strictly convex and

$$\tfrac{2n}{n+2} < p \le q < p + 1.$$

These variational solutions turn out to be unique. Moreover, if the gap satisfies the natural stronger assumption

$$2 \le p \le q < p + \min\left\{1, \tfrac{4}{n}\right\}$$

we show that the spatial derivative Du satisfies the higher integrability

$$u \in L^q_{\mathrm{loc}}\big(0, T; W^{1,q}_{\mathrm{loc}}(\Omega, \mathbb{R}^N)\big)$$

and therefore variational solutions are actually weak solutions. This is joint work with Frank Duzaar and Paolo Marcellini.