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Title: The Dirichlet problem for p-harmonic function on $\mathbf{R}^n$ and metric spaces

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Abstract: *p-harmonic functions* are continuous solutions of the p-Laplace equation

$$\operatorname{div}(|\nabla u|^{p-2}\nabla u) = 0$$

(understood in the weak or distributional sense), where $1 < p < \infty$. The 2-harmonic functions are just the usual harmonic functions, while p-harmonic functions, $p \neq 2$, are the standard prototype for a nonlinear elliptic problem.

The definition of p-harmonic functions has been generalized to many different situations: e.g. weighted $\mathbf{R}^n$, manifolds, graphs and Heisenberg groups. A decade ago they started to be studied on general metric spaces, assuming some suitable assumptions, which unites and extends all the previous theories. In this situation there are no directions and hence no derivatives, but the theory of *upper gradients* turns out to be sufficient to extend the theory of p-harmonic functions to metric spaces.

I will start with discussing how p-harmonic functions are defined on metric spaces. Since Sobolev spaces play an important role in the theory of p-harmonic functions, I will also discuss how Sobolev spaces (Newtonian spaces) are defined on metric spaces. The definition has some advantages to the usual $\mathbf{R}^n$ definition of Sobolev spaces, and is thus of interest also for those just interested in $\mathbf{R}^n$.

I will then turn to the *Dirichlet problem*, i.e. prescribe a boundary function $f : \partial\Omega \to \mathbf{R}$ and try to find a p-harmonic function in Ω with these prescribed boundary values. I will mainly discuss the *Perron method* which makes it possible to solve the Dirichlet problem for noncontinuous f. In particular, I will describe some recent resolutivity results, i.e. results saying that the upper and lower Perron solutions agree so that the Perron method gives one reasonable solution. These results are new also for $\mathbf{R}^2$.