Stability analysis of asymptotic profiles for fast diffusion

Goro Akagi
Kobe University
akagi@port.kobe-u.ac.jp

Abstract

Let Ω be a bounded domain of $\mathbb{R}^N$ with smooth boundary $\partial\Omega$. We are concerned with the Cauchy-Dirichlet problem for fast diffusion equations of the form

$$\partial_t \left(|u|^{m-2}u\right) = \Delta u \qquad \text{in } \Omega \times (0,\infty), \tag{1}$$

$$u = 0 \qquad \text{on } \partial\Omega \times (0,\infty), \tag{2}$$

$$u(\cdot, 0) = u_0 \qquad \text{in } \Omega, \tag{3}$$

where $\partial_t = \partial/\partial t$, $m \in (2, 2^*)$ with $2^* := 2N/(N-2)_+$ and u_0 could be sign-changing. It is well known that every solution $u = u(x,t)$ of (1)–(3) vanishes in finite time at a power rate, and moreover, asymptotic profiles of such vanishing solutions have been studied by many authors.

This talk is concerned with stability analysis for asymptotic profiles of vanishing solutions. We first formulate the notions of stability and instability of (possibly sign-changing) asymptotic profiles and present some stability criteria by investigating fast diffusion flows on an implicit surface in an energy space.

Furthermore, we also discuss annular domain cases, which do not fall within the criteria, by developing some perturbation method for radial symmetric profiles.