Recent advances in parabolic variational inequalities with weakly time-dependent constraints

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In this talk, the well-posedness for parabolic variational inequalities with weakly time-dependent constraints is discussed.

Let us consider the parabolic variational inequalities of the form:

$$u'(t) + \partial\varphi^t(u(t)) \ni f(t) \quad \text{in } H \quad \text{for a.a. } t \in (0,T), \tag{1}$$

in the framework of abstract evolution equations governed by the subdifferentials $\partial\varphi^t$ of time-dependent convex functionals φ^t on a Hilbert space H. The case of $\varphi^t = \varphi + I_{K(t)}$ is mainly treated, where φ is a time-independent convex functional and $I_{K(t)}$ is the indicator function of time-dependent convex sets $K(t)$. We can see the many results, with respect to (1), for example, [1, 2, 5, 6, 7, 9, 11] from various points of view. It seems that some time-regularity assumption on the mapping $t \mapsto \varphi^t$ is necessary in order for problem (1) to possess strong solutions in H. However, this type of regularity assumption is sometimes too strong, for instance, when we apply the result on (1) to a quasi-variational evolution problem of the form:

$$u'(t) + \partial\varphi^t\big(u; u(t)\big) \ni f(t) \quad \text{in } H \quad \text{for a.a. } t \in (0,T), \tag{2}$$

which arises often in various free boundary problems, see the abstract results [3, 10]; in this formulation (2) we are given a family $\{\varphi^t(v;\cdot)\}_{t\in[0,T]}$ of convex functionals $z \mapsto \varphi^t(v;z)$ with parameters $t \in [0,T]$ and v in a subset of $C([0,T];H)$. We mean by (2) to find such a function $u : [0,T] \to H$ that u is the parameter to determine the convex functional $\varphi^t(u;\cdot)$ and at the same time u is a solution of (2) (see [4] for the detail formulation). In most of interesting applications arising in free boundary problems, it is not easy to verify the time regularity of $t \mapsto \varphi^t(u;\cdot)$ which ensures the strong solvability of (2). Therefore, it is worthwhile to discuss the well-posedness of (1) generated by $\partial\varphi^t$ with a weaker time-dependence $t \mapsto \varphi^t$.

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References

[1] M. Biroli, Sur les inéquations paraboliques avec convexe dépendant du temps: solution forte et solution faible, Riv. Mat. Univ. Parma (3), **3**(1974), 33–72.

[2] H. Brézis, Un problème d'évolution avec contraintes unilatérales dépendant du temps, C. R. Acad. Sci. Paris, **274**(1972), 310–313.

[3] R. Kano, Y. Murase and N. Kenmochi, Nonlinear evolution equations generated by subdifferentials with nonlocal constraints, pp. 175–194 in **Vol.86**, Banach Center Publ., 2009.

[4] R. Kano, N. Kenmochi and Y. Murase, Parabolic quasi-variational inequalities with non-local constraints, Adv. Math. Sci. Appl., **19**(2009), 565–583.

[5] N. Kenmochi, Some nonlinear parabolic variational inequalities, Israel J. Math., **22**(1975), 304–331.

[6] N. Kenmochi, Solvability of nonlinear evolution equations with time-dependent constraints and applications, Bull. Fac. Edu., Chiba Univ., **30**(1981), 1–87.

[7] M. Kubo, Characterization of a class of evolution operators generated by time-dependent subdifferentials, Funkcial. Ekvac., **32**(1989), 301–321.

[8] F. Mignot and J. P. Puel, Inéquations d'evolution paraboliques avec convexes dépendant du temps. Applications aux inéquations quasi-variationnelles d'evolution, Arch. Rational Mech. Anal., **64**(1977), 59–91.

[9] J. J. Moreau, Evolution problem associated with a moving convex set in Hilbert space, J. Differential Equations, **26**(1977), 347–374.

[10] U. Stefanelli, Nonlocal quasivariational evolution problems, J. Differential Equations, **229**(2006), 204–228.

[11] Y. Yamada, On evolution equations generated by subdifferential operators, J. Fac. Sci. Univ. Tokyo Sect. IA Math., **23**(1976), 491–515.