

A variational approach to the total variation flow

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In this talk we introduce a purely variational approach to the total variation flow, yielding the existence of global parabolic minimizers, in the sense that

$$\int_0^T \left[\int_{\Omega} u \partial_t \varphi \, dx + \|Du\|(\Omega) \right] dt \leq \int_0^T \|D(u + \varphi)\|(\Omega) \, dt,$$

whenever $T > 0$ and $\varphi \in C_0^\infty(\Omega \times (0, T))$. These evolutionary variational solutions are obtained as limits of maps, minimizing a convex variational functional in $\mathbb{R}^{n+1}$. This approach yields a new way of proving the existence of global weak solutions of the total variation flow

$$\partial_t u - \operatorname{div} \left(\frac{\nabla u}{|\nabla u|} \right) = 0 \quad \text{in } \Omega \times (0, \infty).$$

We are also able to deal with a lower order term, which naturally arises in image restoration problems.

This is joint work with Verena Bögelein (Erlangen), Juha Kinnunen (Helsinki) and Paolo Marcellini (Florence).