Numerical Methods in Engineering Sciences
9/2/2024

First name: _____
Last name: _____

Student ID: _____

☐ I want to take the BASIC EXAM (maximum grade is 24/30)
☐ I want to take the ADVANCED EXAM (maximum grade is 30/30 cum laude)

Total time is 1 hour.

BASIC EXAM

1. Compute the regression parabola $r(x) = c_0 + c_1 x + c_2 x^2$ for the following set of points

$$(-1, 3),\ (0, -5),\ (1, 10),\ (2, 3),\ (3, -1).$$

Report the intermediate computations.

2. Write the pseudo-code of the Implicit Euler Method. Then, given the Cauchy problem

$$\begin{cases} y'(t) = 3 - 2ty(t) & \text{for } t > 0 \\ y(0) = 1 \end{cases}$$

compute two steps of the Implicit Euler Method, with $\Delta t = 1/2$, in order to approximate $y(1)$. Report the intermediate computations.

ADVANCED EXAM

3. Write the pseudo-code of the backward substitution method used to solve linear systems where the matrix is upper triangular. Describe (with full justification) its computational cost. Show how the backward substitution works when solving the system $Ux = b$, with

$$U = \begin{bmatrix} -5 & 0 & 5 \\ 0 & 2 & 3 \\ 0 & 0 & 3 \end{bmatrix}, \qquad b = \begin{bmatrix} 5 \\ 25 \\ 15 \end{bmatrix}$$

4. Describe the composite midpoint quadrature rule, write its pseudo-code, state and prove an error bound.