Numerical Methods in Engineering Sciences
24/6/2024

First name: _____
Last name: _____

Student ID: _____

☐ I want to take the BASIC EXAM (maximum grade is 24/30)
☐ I want to take the ADVANCED EXAM (maximum grade is 30/30 cum laude)

Total time is 1 hour.

BASIC EXAM

1. Apply the Gaussian elimination method, without pivoting, to solve the linear system $Ax = b$, where

$$A = \begin{bmatrix} -4 & 2 & 3 \\ 0 & -4 & -2 \\ 4 & -10 & -6 \end{bmatrix}, \qquad b = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$$

Report the intermediate steps.

2. Write the pseudo-code of the composite trapezoidal quadrature rule, then use the composite trapezoidal quadrature rule to compute an approximation of

$$\int_0^2 \frac{t}{t+1} dt$$

by splitting the integration interval $[0, 2]$ into four uniform subintervals. Report the intermediate computations.

ADVANCED EXAM

3. Describe the Crank-Nicolson scheme for the solution of an ODE and explain its relation with the trapezoidal quadrature rule. Then, compute two steps of the Crank-Nicolson scheme for the problem

$$\begin{cases} y'(t) = -2ty & \text{for } t > 0 \\ y(0) = 1 \end{cases}$$

selecting $\Delta t = 1/2$.

4. Write the pseudo-code of the Jacobi splitting method and describe its computational cost. Prove that if A is a diagonally dominant matrix, then the Jacobi method converges to the exact solution of the system $Ax = b$.