Numerical Methods in Engineering Sciences
24/2/2023

Written Exam

First name: _____

Last name: _____

Student ID: _____

☐ I want to take the BASIC EXAM
☐ I want to take the ADVANCED EXAM

Exam rules:

- Basic exam: the maximum grade is 24/30.

- Advanced exam: the maximum grade is 30/30 cum laude.

Total time is 1 hour. Students who get a positive grade in the written part (i.e., at least 18/30) *might* choose to take an oral exam. For students who choose the basic written exam, the maximum grade obtainable can never exceed 24/30.

BASIC EXAM

1. Given the Cauchy problem:
$$\begin{cases} y'(t) = -2ty(t) + 2t^3 \text{ for } t > 0 \\ y(0) = -1; \end{cases}$$
   compute two steps by the implicit Euler method, with $\Delta t = 1$, in order to approximate $y(2)$. Report the intermediate computations.

2. Describe the Crank-Nicolson scheme for the solution of an ODE and explain its relation with the trapezoidal quadrature rule. Then, compute one step of the Crank-Nicolson scheme for the problem

$$\begin{cases} y'(t) = 2t\left(1 - y(t)\right) \\ y(0) = 3 \end{cases}$$

   selecting $\Delta t = 1$.

ADVANCED EXAM

3. Write the pseudocode of the Gaussian elimination method, without pivoting, and apply it to solve the linear system $Ax = b$, where
$$\begin{bmatrix} 2 & 3 & 1 \\ 1 & 1 & 2 \\ 2 & 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ 2 \end{bmatrix}$$
   showing the intermediate computations.

4. Introduce the power method for the computation of the dominant eigenvalue and eigenvector, with pseudocode. State and prove its convergence. Then, introduce and motivate the inverse power method, reporting its pseudocode.