Numerical Methods in Engineering Sciences
2/2/2023

Written Exam

First name: _____

Last name: _____

Student ID: _____

☐ I want to take the BASIC EXAM
☐ I want to take the ADVANCED EXAM

Exam rules:

- Basic exam: the maximum grade is 24/30.

- Advanced exam: the maximum grade is 30/30 cum laude.

Total time is 1 hour. Students who get a positive grade in the written part (i.e., at least 18/30) *might* choose to take an oral exam. For students who choose the basic written exam, the maximum grade obtainable can never exceed 24/30.

BASIC EXAM

1. Apply the Gaussian elimination method, without pivoting, to solve the linear system $Ax = b$, where

$$\begin{bmatrix} 2 & 4 & 10 \\ 2 & 6 & 20 \\ 1 & 4 & 18 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -6 \\ 12 \\ 18 \end{bmatrix}$$

showing the intermediate computations.

2. Write the pseudocode of the Newton method. With initial value $x_0 = 1$ apply two Newton iteration to the equation

$$(x - 3)^2 = 0,$$

in order to compute its approximate root.

ADVANCED EXAM

3. Write the pseudocode of the backward substitution method used to solve linear systems where the matrix is upper triangular. Describe (with full justification) its computational cost. Show how the backward substitution works when solving the system $Ax = b$, with

$$A = \begin{bmatrix} 3 & 4 & 2 \\ 0 & 3 & -1 \\ 0 & 0 & 2 \end{bmatrix}, \qquad b = \begin{bmatrix} 20 \\ 18 \\ -6 \end{bmatrix}.$$

4. State and prove the theorem on the existence and uniqueness of the Lagrange interpolant of a given function.