Numerical Methods in Engineering Sciences
19/7/2024

First name: _____
Last name: _____

Student ID: _____

☐ I want to take the BASIC EXAM (maximum grade is 24/30)
☐ I want to take the ADVANCED EXAM (maximum grade is 30/30 cum laude)

Total time is 1 hour.

BASIC EXAM

1. Compute the regression line $r(x) = c_0 + c_1 x$ for the following set of points:

$$(-4, -6),\ (-3, 11),\ (-1, 3),\ (0, 5),\ (1, 6),\ (2, 6)$$

2. Write the pseudo-code of the backward substitution method used to solve linear systems where the matrix is upper triangular. Describe (with full justification) its computational cost. Apply the backward substitution method to solve the system $Ux = b$, with

$$U = \begin{bmatrix} 1 & 2 & -2 \\ 0 & 3 & 5 \\ 0 & 0 & -2 \end{bmatrix}, \qquad b = \begin{bmatrix} -6 \\ 7 \\ -4 \end{bmatrix}$$

ADVANCED EXAM

3. Write the pseudo-code of the power method for the computation of the dominant eigenvalue and eigenvector. State conditions that guarantee its convergence. Starting from $v^{(0)} = [-2, -3]^\top$, compute two iterations of the power method on the matrix

$$A = \begin{bmatrix} 3 & -2 \\ -2 & 6 \end{bmatrix}.$$

4. Write the pseudo-code of the Newton method for solving a non-linear equation. State and prove its order of convergence.