EXERCISE 1: Matlab implementation

- Write a function that, given as input an upper triangular matrix U and a vector b, solves the system $Ux = b$ using the backsubstitution method and test it on the system

$$\begin{bmatrix} 3 & -1 & 2 \\ 0 & 1 & -5 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ -4 \\ 2 \end{bmatrix}$$

  whose exact solution is $x_1 = x_2 = x_3 = 1$
- Write a function that, given as input a matrix A and a vector b, solves the system $Ax = b$ using Gaussian elimination

EXERCISE 2: GEM by hands and on Matlab

- Using the Gauss elimination method, solve, by pencil and paper, the linear system below; then check by Matlab the solution (with your own GEM function and with the Matlab command: `A\b`).

$$\begin{bmatrix} 4 & 0 & 12 \\ -2 & 6 & -3 \\ 1 & 2 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

- Same as before, with

$$\begin{bmatrix} -5 & 3 & 4 \\ 10 & -8 & -9 \\ 15 & 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \\ 1 \end{bmatrix}$$

- Same as before, with

$$\begin{bmatrix} 1 & -3 & 4 \\ -1 & 5 & -3 \\ 4 & -8 & 23 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 12 \\ -12 \\ 58 \end{bmatrix}$$