Exercises NMES - 15.1.2025

Exercise 1

Write the LU factorization, without pivoting, of:

$$\begin{matrix}(r_1)\\(r_2)\\(r_3)\end{matrix}\begin{bmatrix}2 & 4 & 4\\1 & 5 & 7\\3 & 12 & 18\end{bmatrix}$$

showing the intermediate computations.

Exercise 2

Compute the linear regression $r(x) = c_0 + c_1 x$ for the set of points

$$(-3, 0),\ (-2, 0),\ (-1, 0),\ (1, 1),\ (2, 2),\ (3, 4).$$

Exercise 3

Starting from $x^{(0)} = \begin{bmatrix}0\\0\\0\end{bmatrix}$, compute 2 iterations of the Jacobi method applied to the system $Ax = b$, where

$$A = \begin{bmatrix}2 & 0 & 1\\0 & 2 & 0\\1 & 0 & 2\end{bmatrix} \qquad b = \begin{bmatrix}1\\0\\1\end{bmatrix}$$

Exercise 4

Apply two bisection iterations to solve the equation

$$x^3 + 3x - 2 = 0 \qquad in\,[0, 1].$$

Exercise 5

With initial guess $\underline{x}^{(0)} = [1, 1]^T$ apply one Newton iteration to find an approximate solution of the system

$$\underline{F}(\underline{x}) = \begin{bmatrix}x_1^2 - 2x_1 + x_2 + 7\\2x_1 - x_2 + 2\end{bmatrix}.$$

Exercise 6

Given the function $f(x) = \cos(2\pi\, x)$ compute its Lagrange interpolant of degree 2 through the points $x_1 = 0$, $x_2 = 1/2$, $x_3 = 1$.

Exercise 7

Describe the Crank-Nicolson scheme for the solution of an ODE and explain its relation with the trapezoidal quadrature rule. Then, compute one step of the Crank-Nicolson scheme for the problem

$$\begin{cases} y'(t) = 2t\,(1 - y(t)) \\ y(0) = 3 \end{cases}$$

selecting $\Delta t = 1$.