

Exercise: ODE solvers

Consider the initial-value problem

$$y'(t) = -3y(t) + 2t - 2, \quad 0 \leq t \leq 1, \quad y(0) = -1.$$

- Approximate $y(1)$ by two steps of implicit Euler ($\Delta t = 1/2$).
- Implement explicit Euler, implicit Euler, Crank–Nicolson and Heun's method. Check your hand calculation using the implementation.
- Plot the four numerical solutions for $\Delta t = 0.5, 0.2, 0.1$, together with

$$y(t) = \frac{2t}{3} - \frac{1}{9}e^{-3t} - \frac{8}{9}.$$

- Compare endpoint errors under mesh refinement. Which methods have first order and which have second order?