

Exercise: integration over a triangulation

Let $\Omega = \{(x, y) : x^2 + y^2 < 1\}$. Construct a triangulation of an inscribed polygon Ω_h , refining both its boundary and interior as $h \rightarrow 0$.

- For example, use MATLAB `de1aunay` on interior points and points sampled on the unit circle. Plot the mesh and check its quality.
- For triangle T with vertices a, b, c , compute

$$|T| = \frac{1}{2} |\det[b - a, c - a]|.$$

Sum the areas and compare with $|\Omega| = \pi$.

- Approximate $\int_{\Omega} f$ by $\sum_T |T| f((a + b + c)/3)$. Test $f = 1$ and $f = x^2 + y^2$, whose exact integrals are π and $\pi/2$.
- Separate the quadrature error on Ω_h from the geometric error due to $\Omega_h \neq \Omega$.

Report how the mesh is constructed and how its maximum triangle diameter depends on the resolution parameter.