

Exercise 1: composite quadrature and convergence

Implement composite midpoint, trapezoidal and Simpson rules on a uniform partition of $[0, 1]$ with $h = 1/N$, and approximate

$$\int_0^1 \sin(\pi x) dx = \frac{2}{\pi}.$$

- Use $N = 4, 8, 16, \dots$; for Simpson's rule, group pairs of subintervals, so that N must be even.
- Plot the absolute error against h on logarithmic axes, together with reference curves proportional to h^2 and h^4 .
- Estimate the order using $\log(E(h)/E(h/2))/\log 2$. Explain the expected orders 2, 2, 4 and stop the comparison before roundoff dominates.

Exercise 2: midpoint and trapezoidal rules by hand

Let $f(x) = 9x^4$ on $[-1, 1]$, with nodes

$$x_0 = -1, \quad x_1 = -\frac{1}{3}, \quad x_2 = \frac{1}{3}, \quad x_3 = 1, \quad h = \frac{2}{3}.$$

- Compute the composite midpoint and trapezoidal approximations.
- Compare them with the exact integral $18/5$.

Midpoint solution. The quadrature points are $-2/3, 0, 2/3$; hence

$$Q_M = \frac{2}{3} \left(\frac{16}{9} + 0 + \frac{16}{9} \right) = \frac{64}{27}.$$

Compute Q_T and both errors yourself.