

Exercise 1: bisection and Newton iteration

Write `[x,its] = bisection(f,a,b,tol,maxits)` and `[x,its] = newton(f,derf,x0,tol,maxits)`.

- Bisection: check continuity and opposite signs at the endpoints; accept an endpoint root. Use the half-width of the current bracket as an error bound.
- Newton: handle a zero derivative and report failure at the iteration limit. Monitor both step size and residual.
- Solve $e^x + \cos x - 2 = 0$, with known root $x^* = 0$. For example, use $[a, b] = [-0.5, 1]$ and $x_0 = 1$. Plot $|x_k - x^*|$ against k .
- Repeat for $e^x - \cos x - 2 = 0$ on $[0, 2]$, starting Newton at $x_0 = 1$. Use a high-accuracy reference, e.g. `fzero(@(x) exp(x)-cos(x)-2, [0,2])`.

Exercise 2: when Newton does not converge

Solve $f(x) = \sqrt[3]{x - \pi} = 0$ using both methods. Use `nthroot(x-pi,3)` for real cube roots, the bracket $[3, 4]$ and a Newton starting point $x_0 \neq \pi$.

- Plot $|x_k - \pi|$ against k and compare the two methods.
- For $x \neq \pi$, derive

$$f'(x) = \frac{1}{3|x - \pi|^{2/3}}, \quad x_{k+1} = 3\pi - 2x_k.$$

Deduce $e_{k+1} = -2e_k$, where $e_k = x_k - \pi$.

- Explain why Newton's local convergence theorem does not apply: f is not differentiable at the root. Stop immediately if an exact root is encountered, before evaluating the derivative.