

# Exercise 1: the power method

Write `[v,lambda] = power(A,maxit,tol)` using a random initial vector, normalization in the Euclidean norm, and the Rayleigh quotient.

- Test on `Q = orth(randn(10,10)); A = Q*diag(1:10)*Q'`. Order the eigenvalues as  $\lambda_1 = 10, \dots, \lambda_{10} = 1$ .
- Plot  $|\lambda_1 - \lambda^{(k)}|/|\lambda_1|$  and  $(9/10)^k$  against  $k$ . For this symmetric matrix, also compare with  $(9/10)^{2k}$ : explain the different rates for eigenvector and Rayleigh-quotient errors.
- Repeat with `Q = randn(10,10); A = (Q*diag(1:10))/Q`. Check that  $Q$  is nonsingular and inspect `cond(Q)`. Compare with the symmetric case.

Stop when  $\|Av - \lambda v\|_2 \leq \text{tol} \|A\|_2 \|v\|_2$ , or when the iteration limit is reached. Report which condition holds; handle a zero iterate before normalizing.

## Exercise 2: shifted inverse iteration

Implement `inverse_power`, solving a linear system with  $A - \sigma I$  at each iteration (use MATLAB backslash; do not form its inverse).

- Use the nonsymmetric matrix from Exercise 1 and approximate  $\lambda_9 = 2$  with shifts

$$\sigma = 1.55, 1.65, 1.75, 1.85, 1.95.$$

- Normalize each iterate and estimate the eigenvalue of  $A$  with its Rayleigh quotient. Plot  $|2 - \lambda^{(k)}|$  against  $k$  for each shift.
- Use a residual test and an iteration limit as in Exercise 1. Reuse a factorization of  $A - \sigma I$  for each fixed shift.
- Optional: plot the iteration count against

$$\frac{|2 - \sigma|}{|1 - \sigma|}$$

and explain its role as an asymptotic convergence factor for the eigenvector. Discuss why the shift must not be an eigenvalue.