

EXERCISE 3: LU factorization

- Write a MATLAB function that, given as input a matrix A , returns the L and U factors of the LU factorization without pivoting and test it on the following matrices:

$$\begin{bmatrix} 4 & 0 & 12 \\ -2 & 6 & -3 \\ 1 & 2 & 5 \end{bmatrix}$$

$$\begin{bmatrix} -5 & 3 & 4 \\ 10 & -8 & -9 \\ 15 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -3 & 4 \\ -1 & 5 & -3 \\ 4 & -8 & 23 \end{bmatrix}$$

EXERCISE 4: GEM with pivoting

- Write a MATLAB function that, given as input a matrix A and a vector b , solves the system $Ax = b$ using Gaussian elimination with pivoting and test it on the two systems:

$$\begin{bmatrix} 4 & 0 & 12 \\ -2 & 6 & -3 \\ 1 & 2 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}; \quad \begin{bmatrix} 2 & 2 & 0 \\ 1 & 1 & -1 \\ 3 & -2 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 5 \end{bmatrix}$$

whose exact solutions are respectively $x = [-3, -1/2, 1]^T$ and $x = [1, 1, 1]^T$.

EXERCISE 5: solving linear systems and more...

- Using the LU factorization function above, write a MATLAB function that returns the inverse of an input matrix.
- Modify the above function to compute the determinant of an input matrix A , and test it on the matrices in the previous slide. Finally, check the results using MATLAB command **det**
- Solve a system $Ax = f$ with user-made functions above, where f arbitrary chosen and

$$A = \begin{bmatrix} 2 & -1 & 0 & \dots & 0 \\ -1 & 2 & -1 & \dots & 0 \\ 0 & \ddots & \ddots & \ddots & 0 \\ \vdots & & -1 & 2 & -1 \\ 0 & 0 & \dots & -1 & 2 \end{bmatrix}$$

In particular, compute x with either the GEM/LU methods implemented above and by explicitly computing $x = A^{-1}f$ (i.e., computing A^{-1} with the function above). Using the MATLAB commands **tic** and **toc**, measure the time required by the two approaches to compute the solution. The matrix size should be chosen large enough so that the time difference is relevant.

EXERCISE 6: LU with pivoting (optional)

- Write a MATLAB function that, given as input a matrix A , returns the P , L and U factors of the LU factorization with pivoting and test it on the following matrix:

$$\begin{bmatrix} 2 & 2 & 0 \\ 1 & 1 & -1 \\ 3 & -2 & 4 \end{bmatrix}$$