

EXERCISE 1: MATLAB implementation

- Write a function that, given as input an upper triangular matrix U and a vector b , solves the system $Ux = b$ using the back substitution method and test it on the system

$$\begin{bmatrix} 3 & -1 & 2 \\ 0 & 1 & -5 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ -4 \\ 2 \end{bmatrix}$$

whose exact solution is $x_1 = x_2 = x_3 = 1$

- Write a function that, given as input a matrix A and a vector b , solves the system $Ax = b$ using Gaussian elimination

EXERCISE 2: GEM by hand and on MATLAB

- Using the Gaussian elimination method, solve, by pencil and paper, the linear system below; then check by MATLAB the solution (with your own GEM function and with the MATLAB command: $A \backslash b$).

$$\begin{bmatrix} 4 & 0 & 12 \\ -2 & 6 & -3 \\ 1 & 2 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

- Same as before, with

$$\begin{bmatrix} -5 & 3 & 4 \\ 10 & -8 & -9 \\ 15 & 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \\ 1 \end{bmatrix}$$

- Same as before, with

$$\begin{bmatrix} 1 & -3 & 4 \\ -1 & 5 & -3 \\ 4 & -8 & 23 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 12 \\ -12 \\ 58 \end{bmatrix}$$

Exercise 2: solution of system 1

Gaussian elimination gives the equivalent triangular system

$$\begin{bmatrix} 4 & 0 & 12 \\ 0 & 6 & 3 \\ 0 & 0 & 1 \end{bmatrix} x = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

Back substitution then yields

$$x = \begin{bmatrix} -3 \\ -\frac{1}{2} \\ 1 \end{bmatrix}.$$

Check the result by substituting it into the original system.

Exercise 2: solution of system 2

Gaussian elimination gives the equivalent triangular system

$$\begin{bmatrix} -5 & 3 & 4 \\ 0 & -2 & -1 \\ 0 & 0 & 9 \end{bmatrix} x = \begin{bmatrix} 1 \\ 7 \\ 39 \end{bmatrix}.$$

Back substitution then yields

$$x = \begin{bmatrix} -\frac{2}{15} \\ -\frac{17}{3} \\ \frac{13}{3} \end{bmatrix}.$$

Check the result by substituting it into the original system.

Exercise 2: solution of system 3

Gaussian elimination gives the equivalent triangular system

$$\begin{bmatrix} 1 & -3 & 4 \\ 0 & 2 & 1 \\ 0 & 0 & 5 \end{bmatrix} x = \begin{bmatrix} 12 \\ 0 \\ 10 \end{bmatrix}.$$

Back substitution then yields

$$x = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}.$$

Check the result by substituting it into the original system.