Nonlocal temperature-dependent phase-field models for non-isothermal phase transitions

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This is a joint work with Pavel Krejčí and Jürgen Sprekels (Weierstrass Institute for Applied Analysis and Stochastics, Berlin). We propose a model for non-isothermal phase transitions with non-conserved order parameter driven by a spatially nonlocal free energy with respect to both the temperature and the order parameter.

For $(x,t) \in Q_T$, where $Q_T := \Omega \times (0,T)$ and Ω is the physical body in which the phase transition occurs, the system of equations is

$$
\begin{aligned}
& c_V \vartheta_t(x) - 2\vartheta(x) \int_\Omega K_{\tau\tau}(\tau,x,y)\big|_{\tau=\vartheta(x)+\vartheta(y)} (\vartheta_t(x) + \vartheta_t(y))\, G(\chi(x) - \chi(y))\, dy \\
& \quad = \kappa \Delta \vartheta(x) + 2\vartheta(x) \int_\Omega K_\tau(\tau,x,y)\big|_{\tau=\vartheta(x)+\vartheta(y)} G'(\chi(x) - \chi(y))(\chi_t(x) - \chi_t(y))\, dy \\
& \qquad - (\lambda(\chi(x)) + \beta\varphi(\chi(x)))_t - 2\chi_t(x) \int_\Omega K(\vartheta(x) + \vartheta(y), x, y) G'(\chi(x) - \chi(y))\, dy\,, \\
& \mu(\vartheta(x))\chi_t(x) + \vartheta(x)\sigma'(\chi(x)) + \lambda'(\chi(x)) \\
& \qquad + 2\int_\Omega K(\vartheta(x) + \vartheta(y), x, y) G'(\chi(x) - \chi(y))\, dy \in -(\beta + \vartheta(x))\partial\varphi(\chi(x)),
\end{aligned}
$$

which we couple with suitable boundary and initial conditions.

Here Δ is the Laplace operator, the subscripts $_t$ and $_\tau$ denote partial derivatives, $\kappa > 0$ is a constant which stands for the heat conductivity, $c_V > 0$ is the specific heat, σ and λ are smooth functions describing the local dependence on χ of the entropy and of the latent heat, respectively, φ is a general proper, convex, and lower semicontinuous function, $\beta > 0$ is a constant parameter, $K : \mathbf{R}^+ \times \Omega \times \Omega \to \mathbf{R}$ is a sufficiently regular symmetric kernel describing nonlocal interactions, and G is an even smooth function having some boundedness properties on the domain of φ.

We show that this system turns out to be thermodynamically consistent and to admit a strong solution.