UNIFORMLY ELLIPTIC OPERATORS WITH GENERALIZED WENTZELL BOUNDARY CONDITIONS

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Let $\Omega \subset \mathbf{R}^n$ be an open bounded domain with smooth boundary $\partial\Omega$. Consider a second order uniformly elliptic operator

$$\nabla\cdot(a\nabla)$$

equipped with generalized Wentzell boundary conditions of the type

$$\nabla\cdot(a\nabla u) + <a\nabla u, \nu> + \gamma u = 0 \qquad \text{on } \partial\Omega,$$

for u regular enough, under suitable assumptions on the matrix of coefficients $a = (a_{ij})$.

For such an operator we consider a suitable realization acting on the product space $X_2 := L^2(\Omega) \times L^2(\partial\Omega)$. By methods based on the theories of Dirichlet sesquilinear forms and of ultracontractive submarkovian semigroups we show that such a realization generates a cosine operator function on X_2 and thus an analytic semigroup of angle $\frac{\pi}{2}$ as well. This semigroup is ultracontractive, and in particular it maps X_2 into X_∞ for time $t > 0$. Conclusions about the L^p-well-posedness of the diffusion problem

$$\begin{cases} \dot{u}(t,x) = \nabla\cdot(a\nabla u(t,x)), & t > 0,\ x \in \Omega, \\ \nabla\cdot(a(z)\nabla u(t,z)) + <a(z)\nabla u(t,z), \nu(z)> & \\ \qquad\qquad + \tilde{\gamma}(z)u(t,z) = 0, & t > 0,\ z \in \partial\Omega, \\ u(0,x) = f(x), & x \in \Omega. \end{cases}$$

can thus be drawn. Our results should be compared with those obtained, among others, in [1], [2], and [3].

This is joint work with S. Romanelli [4].

References

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