Study of the Limit of Transmission Problems in a Thin Layer by the Sum Theory of Linear Operators

We consider a family $P(d)$, where d is a small positive parameter, of singular elliptic transmission problems in the juxtaposition $]-1, d[\times G$ of two bodies, the cylindric medium $V =]-1, 0[\times G$ and the thin layer $V(d) =]0, d[\times G$. It is assumed that the coefficient in $V(d)$ is $1/d$. Such problems model for instance heat propagation between the body V, the layer $V(d)$ (when supposed with infinite conductivity) and the ambient space.

In a first step, we perform a rescaling in the thin layer to transform the problem $P(d)$ in a problem $Q(d)$ set in the fixed domain domain $]-1, 1[\times G$. Then we write problem $Q(d)$ in the form of a sum of linear operators and we show that the sum theory developped by Da Prato-Grisvard works. This gives an explicit writing of the strong solution $u(d)$ as a Dunford Integral in the L^p spaces, $p > 1$.

In the second step, we study the behavior of $u(d)$ as d tends to 0. We deduce that the family of solutions $u(d)$ converges in L^p to a function u in the case of second member in L^p and converges in $W^{(1+2s,p)}$ for a second member in $W^{(2s,p)}$ (s in $]0, 1/2[$). Moreover, in virtue of the techniques used in the study of abstract differential equations, we then prove that the restriction of the limit u to $]-1, 0[\times G$ is in fact, the solution to an elliptic problem on $]-1, 0[\times G$, with a boundary condition of Ventcel's type and it has an optimal regularity.

Finally, we go back to our first problem, in order to translate all the above results on the solution $v(d)$ of $P(d)$.