Gradient estimates for solutions to Cauchy-Neumann problems in unbounded and non convex open sets

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In this talk we deal with the Cauchy-Neumann problem

$$\begin{cases} D_t u(t,x) = \mathcal{A}u(t,x), & t>0, \quad x\in\Omega, \\ \dfrac{\partial u}{\partial \nu}(t,x) = 0, & t>0, \quad x\in\partial\Omega, \\ u(0,x) = f(x), & \qquad\;\; x\in\Omega, \end{cases} \tag{1}$$

in smooth and non convex unbounded open sets $\Omega \subset \mathbb{R}^N$. Here, $\mathcal{A}$ is an uniformly elliptic operator with coefficients which are smooth and possibly unbounded in $\overline{\Omega}$, and $f \in C_b(\overline{\Omega})$. Under suitable assumptions on the coefficients of the operator $\mathcal{A}$, we show that the problem (1) admits a unique classical solution u, which is bounded in $[0,T]\times\Omega$ for any $T>0$.

We also show both uniform (with respect to the sup-norm in Ω) and pointwise gradient estimates for u. Such estimates can be used to prove some interesting consequences, such as a Liouville-type theorem.

The result discussed in this talk have been obtained in collaboration with M. Bertoldi and S. Fornaro.