Identification of unknown obstacles and boundaries

in a fluide via boundary measurements

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In this talk we consider an inverse problem for the Stokes equations, modeling an incompressible viscous fluid contained in a cavity inside which immersed in the fluid, there is an unknown solid body. More precisely assume that $\Omega \subset \mathbf{R}^N$ is a bounded Lipschitz domain and that $\omega_0 \subset\subset \Omega$ is a Lipschitz subdomain representing an *obstacle*, denote by Γ_1 and Γ_0 two pieces of the boundary such that $\partial\Omega = \Gamma_1 \cup \Gamma_0$ for $\varphi : \Gamma_1 \longrightarrow \mathbf{R}^N$ given such that $\int_{\Gamma_1} \varphi(\sigma) \cdot \mathbf{n}(\sigma) d\sigma$. Then for $\mathbf{u} : \Omega \setminus \omega_0 \longrightarrow \mathbf{R}^N$ and $p : \Omega \setminus \omega_0 \longrightarrow \mathbf{R}$ consider

$$\sigma(\mathbf{u}, p) := \frac{1}{2}(D\mathbf{u} + (D\mathbf{u})^*) - pI,$$

and assume that $(\mathbf{u}, p)$ satisfies the incompressible Stokes equations $\text{div}(\mathbf{u}) = 0$ and

$$\text{div}(\sigma(\mathbf{u}, p)) = 0 \ \text{ in } \ \Omega \setminus \omega_0, \quad \mathbf{u} = \varphi \ \text{ on } \ \Gamma_1, \quad \mathbf{u} = 0 \ \text{ on } \ \Gamma_0 \cup \partial\omega_0.$$

(Here it is assumed that Γ_1 is some accessible part of the boundary, while part of, or the totality of, Γ_0 is inaccessible). Next we consider the Poincaré-Steklov operator, corresponding to the Cauchy forces exerted on the boundary,

$$\Lambda(\varphi) := \sigma(\mathbf{u}, p)\mathbf{n} \ \text{ on } \ \Gamma \subset \Gamma_1.$$

We are interested in the determination of the shape and location of ω_0 and Γ_0, by means of measurements of the Cauch forces on some part of the exterior boundary. We show that, provided that $\varphi \not\equiv 0$, knowledge of $\Lambda(\varphi)$ implies that of ω_0 and Γ_0. We show also a directional continuity result for the external measures with respect to deformations of the solid body represented by ω_0. Some numerical results are also obtained when the rigid body is for instance a ball.