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Some results on doubly nonlinear parabolic problems

In this talk we will present some recent results, obtained in collaboration with A. Segatti and U. Stefanelli, concerning existence, uniqueness, regularity properties, and long time behavior of solutions to the following *doubly nonlinear* partial differential inclusion:

$$\alpha(u_t) + Bu + W'(u) \ni f. \qquad (1)$$

Relation (1) is settled in a smooth and bounded domain $\Omega \subset \mathbf{R}^3$ for times $t \in (0, +\infty)$ and complemented with initial conditions and homogeneous boundary conditions of either Dirichlet or Neumann type. Indeed, B denotes there a (possibly nonlinear and degenerate) elliptic operator, while $\alpha \subset \mathbf{R} \times \mathbf{R}$ is a maximal monotone graph and W' represents the derivative of a potential on u, convex in its principal part, but possibly *singular*, i.e. identically $+\infty$ outside a bounded set. Finally, f is a source term.

It was proved by Colli and Visintin in 1990 that the initial-boundary value problem for (1) admits at least one global solution in a Hilbert setting under suitable assumptions on α, B and W. In particular, α was supposed to be a *bounded* operator. These results were then extended by Colli in 1992 to the Banach setting.

Here we will show that, in a regularity framework different from that of Colli and Visintin, there still holds global existence of solutions also for α unbounded. Moreover, we will investigate further properties of the solutions, namely uniqueness, additional regularity, and long time behavior. These properties will be obtained under more restrictive conditions on the operators α, B and on the potential W.