

Ginzburg-Landau equation and phase transitions

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In any phase transition the states of the two phases are endowed with different structures or symmetries. In particular, for many materials below a critical temperature, the order structure is greater than above. In the solid-fluid transition, the solid phase has a greater structure due to the crystal symmetry group, in ferromagnetism below the critical temperature the magnetic moments are aligned, in superconductivity and superfluidity the order is associated with the structure of the super-electron velocity.

As it is well known, the turbulence rises when the velocity of a fluid, for instance inside a cylindrical pipe, exceeds a fixed threshold which is related to the Reynolds number

$$R_e = \frac{dv}{\nu}$$

where v denotes the mean flow down the pipe, ν the viscosity of the fluid and d the pipe diameter. In such a case we observe a transition from laminar to turbulent flow, which is governed by the parameter R_e in a way very similar to the phase transition that we meet in superconductivity. So that there are many analogies between these two phenomena. The most evident is given by the same vortices which we observe in Bénard's convection and in type II superconductors.

Motivated by this view, we shall suggest the study of turbulence as a phase transition of the second order, where the phases are the laminar and turbulent flow by means of a suitable system realized by means of the Ginzburg-Landau and Navier-Stokes equations.