Phase transitions and hysteresis: new perspectives and results

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Supported by the
FP7-IDEAS-ERC-StG Grant“EntroPhase” #256872 (P.I. E. Rocca)

Giornata di Studio *“Prospettive di sviluppo della matematica applicata in Italia 2013”* - Workshop SIMAI Giovani 2013

Roma - March 11, 2013

Plan of the talk

- Hysteresis: a rate-independent memory effect
- The stop and the Prandtl-Ishlinskii operators
- New theory of oscillating elastoplastic beams and plates
- Motivation for material fatigue
- Evolution equation for the fatigue
- The model with phase transition
- Thermodynamical consistency
- Conclusion

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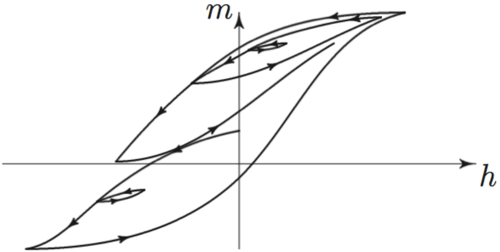
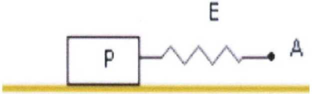
Hysteresis: a rate-independent memory effect

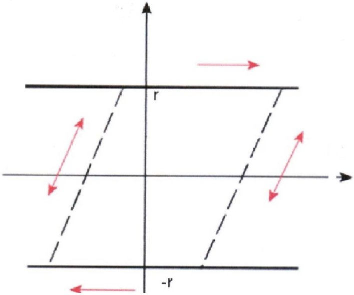
- **Hysteresis:** a rate-independent memory effect (multidisciplinary character)

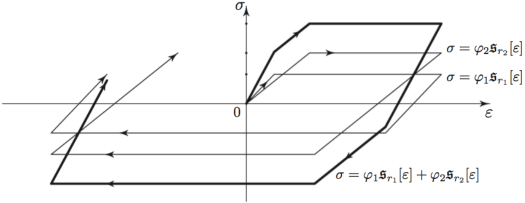


Tipical hysteresis diagram in ferromagnetism (h magnetic field, m magnetization).

The stop and the Prandtl-Ishlinskii operators







A classical hysteresis-type model for 1D elastoplasticity

- Introduced by L. Prandtl and A. Yu. Ishlinskii (extensions to the multidimensional case are possible)
- The relation between (one-dimensional) strain ε and stress σ is given in the form of the so-called Prandtl-Ishlinskii operator

$$\sigma = \mathscr{P}[\varepsilon](t) = \int_0^\infty \mathfrak{s}_r[\varepsilon](t)\,\varphi(r)\,\mathrm{d}r$$

  for all $\varepsilon \in W^{1,1}(0,T)$. Here $\varphi > 0$ is a nonnegative weight function not known a priori and $\mathfrak{s}_r$ represents the **one-dimensional elastic-ideally plastic element or stop operator**, with the threshold $r > 0$
- Prandtl-Ishlinskii description of elastoplasticity: a superposition of infinitely many stop operators having different thresholds (very imaginative and easily understood) BUT engineers very often prefer classical engineering approaches like the three-dimensional von Mises or Tresca models

- Motivation: the disadvantage that the weight function φ **is not known a priori** and must be identified

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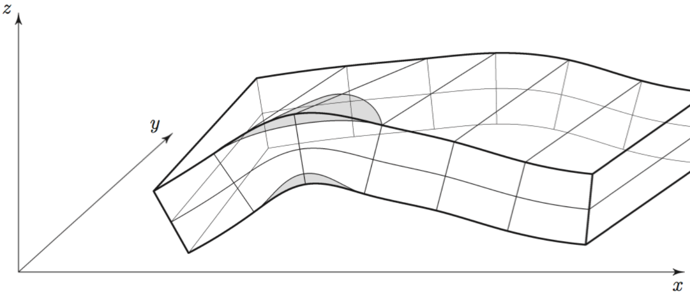

A plate section with grey plasticized zone.

New theory of oscillating elastoplastic beams and plates

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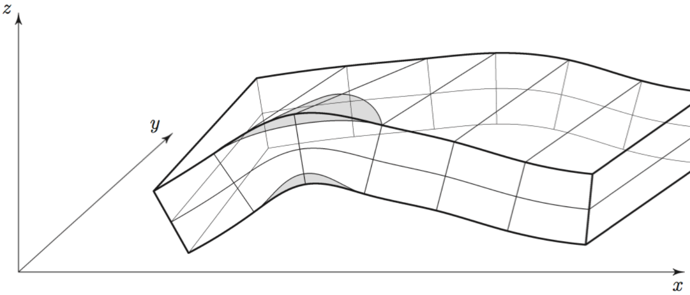

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Motivation for material fatigue

- Plastic deformations lead to energy dissipation and material fatigue, manifested by material softening, heat release, material failure in finite time
- Very important: take into account the effects of energy exchange and estimating the lifetime of oscillating thermoelastoplastic structures under material fatigue
- **Aim:** develop a thermodynamically consistent theory of oscillating thermoelastoplastic plates under material fatigue (dynamic approach - different from literature)
- The resulting system from the theory developed by Krejčí & al:

$$\partial_{tt} w - \partial_{tt} \Delta w + \mathbf{D}_2^* \sigma = g ,$$

$$\sigma = \mathbf{B}\varepsilon + \int_0^\infty \mathfrak{s}_{rZ}[\varepsilon](t)\,\varphi(r)\,\mathrm{d}r$$

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- We introduce $\theta > 0$ (absolute temperature) and $m(x,t) \geq 0$ (material fatigue); **aim:** get an evolution equation for m consistent from the thermodynamic point of view

Evolution equation for the fatigue

- **Main assumption:** proportionality between rate of fatigue $\partial_t m$ and

$$\begin{aligned}\mathscr{D} &= \langle \sigma, \partial_t \varepsilon \rangle - \partial_t \theta \mathscr{S}[\theta, \varepsilon] - \partial_t \mathscr{F}[\theta, \varepsilon] \\ &= -\frac{1}{2} \langle \mathbf{B}'(m) \varepsilon, \varepsilon \rangle \, \partial_t m + \int_0^\infty \langle \partial_t (\varepsilon - \mathfrak{s}_{rZ}[\varepsilon]), \mathfrak{s}_{rZ}[\varepsilon] \rangle \, \varphi(\theta, r) \, \mathrm{d}r\end{aligned}$$

  where $\mathscr{F}$ is the **specific free energy** and $\mathscr{S}$ is the **specific entropy**
- Justified by the so-called **rainflow method for cyclic fatigue accumulation** in uniaxial processes (counts closed hysteresis loops in the loading hystory - mechanism of energy dissipation)
- In multiaxial loading processes? Experimental measurements at the point of material failure: strong temperature increase, manifested by energy dissipation peak (temperature tests are in engineering practice for damage analysis in high frequency regimes (e.g. in aircraft industry))

$$\left(\frac{1}{C(\theta)} + \frac{1}{2} \langle \mathbf{B}'(m) \varepsilon, \varepsilon \rangle \right) \partial_t m = \int_0^\infty \langle \partial_t (\varepsilon - \mathfrak{s}_{rZ}[\varepsilon]), \mathfrak{s}_{rZ}[\varepsilon] \rangle \, \varphi(\theta, r) \, \mathrm{d}r$$

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The model with phase transition

- **Motivation:**
  - possibility to account also for decreasing fatigue rate (in view of engineering applications)
  - the material can be partially repaired by local melting
- **How to achieve this goal:**
- Phase transition equation in the form of melting-solidification law

$$\gamma\chi_t \in -\partial_\chi \mathscr{F}[\varepsilon, \theta, \chi] \qquad \chi \in [0,1]$$

$\chi_0 \in [0,1]$ some initial condition, $A(x,t) := \int_0^t \frac{1}{\gamma}\left(\frac{L}{\theta_c}(\theta - \theta_c)\right)(x,\tau)\,d\tau$

$$\boxed{(\chi_t - A_t)(z - \chi) \geq 0 \text{ for all } z \in [0,1]}$$

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    m: material fatigue and χ degree of melting
    the time of failure of the material can be shifted
    possibly considering a sufficiently large time interval of observation (useful engineering viewpoint) a global solution of the corresponding PDEs system can be found
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$$\boxed{\chi \in \mathfrak{s}_{[0,1]}[\chi_0, A]}$$

$\mathfrak{s}_{[0,1]}$ is a shifted stop

Thermodynamical consistency

- If we introduce $\mathscr{F}[\varepsilon,\theta,\chi]$ **specific free energy**, $\mathscr{S}[\varepsilon,\theta,\chi]$ **specific entropy** and $\mathscr{U}[\varepsilon,\theta,\chi]$ **internal energy** we are able to show that the first and second principles of thermodynamics are satisfied

$$\frac{\partial}{\partial t}\mathscr{U}[\varepsilon,\theta,\chi] + \mathrm{div}\,\mathbf{q} = \langle \sigma, \varepsilon_t \rangle \quad \text{(energy conservation)}$$

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- **Evolution equation for m:**

$$\left(C - \langle \mathbf{B}'(m)\varepsilon, \varepsilon\rangle\right) m_t = -h(\chi_t) + \int_0^\infty \langle \partial_t(\varepsilon - \mathfrak{s}_{rZ}[\varepsilon])\mathfrak{s}_{rZ}[\varepsilon]\rangle \varphi(\theta, r)\,\mathrm{d}r$$

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- Rainflow method for fatigue evaluation in elastoplastic materials (uniaxial cyclic loading) allow to consider dissipated energy as a measure for fatigue
- The solution cannot be expected to exist globally: singularities (thermal shocks) occur in finite time
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Spring School on "Rate-independent evolutions and hysteresis modelling", Milano, May 27-31, 2013
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