Master Program in Electronic Engineering

Advanced Mathematical Methods for Engineers

February 23, 2023

1. Consider, for $a \in \mathbf{R}$, the following Cauchy Problem

$$\begin{cases} y'(x) - \dfrac{x+2}{x^2+1}y + \dfrac{y^2}{(x^2+1)^{3/2}} = 0 \\ y(0) = a^2. \end{cases}$$

a) Discuss local and global existence and uniqueness of solutions, depending on a.

b) Find explicitly the solutions y_a (depending on a).

c) Find the values of the parameter a such that $\mathrm{dom}(y_a) \equiv \mathbf{R}$.

2. Given, for $x, y \geq 0$, the nonlinear ODE system

$$\begin{cases} x' = x - yx - x^2 \\ y' = xy - 2y \end{cases}$$

find the stationary points and discuss their stability.

3. Find

$$\lim_{n \to +\infty} \int_0^n \left(1 + \frac{x}{n}\right)^n e^{-\pi x} \, dx$$

justifying all steps.

4. Find the solutions u in $\mathcal{D}'(\mathbf{R})$ of the equation:

$$(x^2 - 1)u = \delta_0$$

justifying the computations.