Master Program in Electronic Engineering

Advanced Mathematical Methods for Engineers

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1. Let $a \in \mathbf{R}$ and consider the following Cauchy Problem

$$\begin{cases} y'(x) = \dfrac{y(x)}{x-1} + x(y(x))^2 \\ y(0) = a. \end{cases}$$

1.1) Discuss local existence and uniqueness of solutions.

1.2) Find explicitly the solution, depending on a.

1.3) Find the values of a such that the solution is well defined on $(-\infty, 1)$.

2. Given $\alpha \in \mathbf{R}$ and the following ODE system

$$\begin{cases} x' = -3x + \alpha^2 y \\ y' = x - 3y, \end{cases}$$

find the values of α such that the null solution $(0,0)$ is asymptotically stable for the system.

3. Compute the **first and second derivatives in sense of distributions** of the signals

$$u(t) := \operatorname{sign}(\cos(\pi t)),$$
$$v(t) := \begin{cases} -\operatorname{sign}(t) & \text{if } |t| \geq 1 \\ t & \text{if } |t| < 1 \end{cases}$$

where

$$\operatorname{sign}(t) := \begin{cases} 1 & \text{if } t \geq 0 \\ -1 & \text{if } t < 0. \end{cases}$$

4. Using the method of separation of variables, determine the solution of the following Initial-Boundary Value Problem

$$\begin{cases} u_t(x,t) - u_{xx}(x,t) - 2u_x(x,t) = 0 & \text{for } (x,t) \in (0,\ell) \times \mathbb{R}^+ \\ u(x,0) = 2e^{-x} \sin\left(\frac{3\pi x}{\ell}\right) & \text{for } x \in (0,\ell) \\ u(0,t) = u(\ell,t) = 0 & \text{for } t \in \mathbb{R}^+ \end{cases}$$

where $\ell \in \mathbb{R}^+$.