Non-isothermal cyclic fatigue in oscillating elasto-plastic structures with hysteresis

Michela Eleuteri

Università degli Studi di Milano

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Milano, June 5th, 2013

Plan of the seminar

- Theory of hysteresis operators powerful tool for solving mathematical problems in various applications (solid mechanics, material fatigue, ferromagnetism, phase transitions)

Elasto-plastic oscillations of beams and plates with material fatigue

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- Main modeling assumption: proportionality between fatigue rate and dissipation rate
- Influence of energy dissipation (due to material softening on the damage increase) taken into account - fatigue accumulation accelerated - a singularity (material failure) may develop in finite time
- Account also for *decreasing fatigue rate* (phase parameter χ)
- Discuss thermodynamic consistency of the model with a proper choice of the evolution equation for the fatigue parameter m

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- The material fatigue
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- Numerics in PDEs with hysteresis in elastoplasticity?

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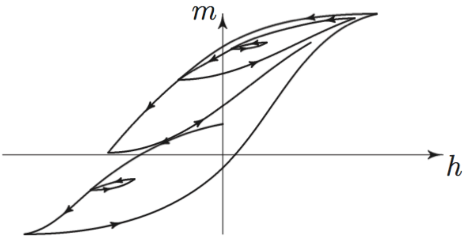
- **Hysteresis:** a rate-independent memory effect (multidisciplinary character)



Tipical hysteresis diagram in ferromagnetism (h magnetic field, m magnetization).

- Hysteresis present not only in ferromagnetism, but also in phase transitions, elastoplasticity, shape memory alloys, magnetostrictive and piezoeletric materials, economy, biology...

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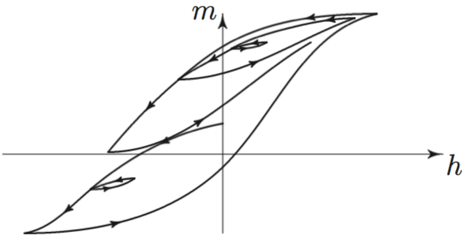

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The stop model

Figure 1: The stop model.

The stop model

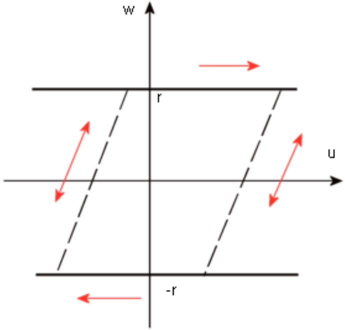

Figure 2: Hysteretic behaviour of the stop model.

- Gven a parameter $r > 0$, a function $\varepsilon : [0,T] \to \mathbb{R}$ and an initial condition $\sigma^0 \in [-r,r]$
- We look for functions $\sigma, \xi : [0,T] \to \mathbb{R}$ such that $\sigma(0) = \sigma^0$ and

$$\begin{aligned} \sigma(t) + \xi(t) &= \varepsilon(t) \\ |\sigma(t)| &\le r \\ \dot{\xi}(t)(\sigma(t) - \tilde{\sigma}) &\ge 0 \quad \forall \tilde{\sigma} \in [-r,r]. \end{aligned}$$

- For all $\varepsilon \in W^{1,1}(0,T)$ and $\sigma^0 \in [-r,r]$, the previous problem admits a unique solution $\sigma \in W^{1,1}(0,T)$
- The map $\mathfrak{s}_r : [-r,r] \times W^{1,1}(0,T) \to W^{1,1}(0,T)$, $\mathfrak{s}_r[\sigma^0, \varepsilon] = \sigma$ is called **stop** or **elasto-plastic element**. $\mathfrak{s}_r : [-r,r] \times C([0,T]) \to C([0,T])$
- Multidimensional extension of the stop model

A classical hysteresis-type model for 1D elastoplasticity

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The multidimensional stop model

- Given a closed convex set Z including the origin in his interior 0 in a real separable Banach space X, a function $\varepsilon : [0,T] \to X$ and an initial condition $\sigma^0 \in Z$, we look for functions $\sigma, \xi : [0,T] \to X$ such that $\sigma(0) = \sigma^0$ and

$$\sigma(x,0) = Q_Z(\varepsilon(x,0))$$

$$\sigma(t) \in Z$$

$$\left\langle \dot{\xi}(t), \sigma(t) - \tilde{\sigma} \right\rangle \ \geq \ 0 \ \ \forall \tilde{\sigma} \in Z.$$

For all $\varepsilon \in W^{1,1}(0,T;X)$ and $\sigma^0 \in Z$, the previous system admits a unique solution $\mathfrak{s}_Z[\sigma^0, \varepsilon] = \sigma \in W^{1,1}(0,T;X)$. The map $\mathfrak{s}_Z : Z \times W^{1,1}(0,T;X) \to W^{1,1}(0,T;X)$ is continuous and admits a continuous extension $\mathfrak{s}_Z : Z \times C([0,T];X) \to C([0,T];X)$ Back

A classical hysteresis-type model for 1D elastoplasticity

- A classical hysteresis-type model for one-dimensional elastoplasticity was introduced by L. Prandtl and A. Yu. Ishlinskii
- In their model, the relation between (one-dimensional) strain ε and stress σ is given in the form of the so-called Prandtl-Ishlinskii operator

$$\sigma = \mathscr{P}[\varepsilon](t) = \int_0^\infty \mathfrak{s}_r[\varepsilon](t)\,\varphi(r)\,\mathrm{d}r$$

for all $\varepsilon \in W^{1,1}(0,T)$. Here $\varphi > 0$ is a nonnegative weight function not known a priori and $\mathfrak{s}_r$ represents the **one-dimensional elastic-ideally plastic element or stop operator**, with the threshold $r > 0$.

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The stop operators and their combinations

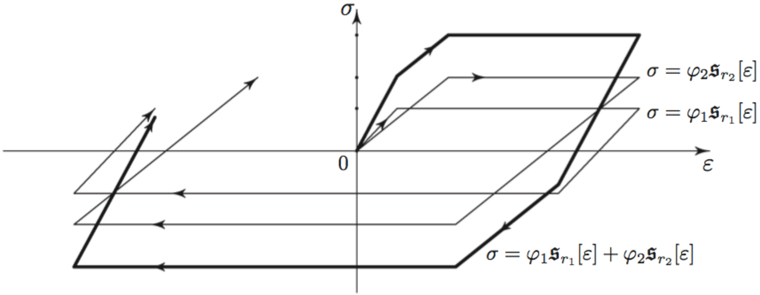

Figure 3: the stop operators and their combinations.

The Prandtl-Ishlinskii model

- ☺ very imaginative and easily understood (superposition of many stops having different thresholds
- ☺ **multi-yield:** describes gradual plasticization process
- ☹ the weight function φ **is not known a priori** and must be identified

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A plate section with grey plasticized zone

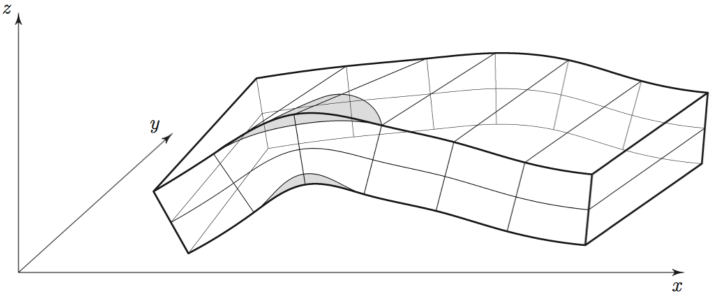

Figure 4: A plate section with grey plasticized zone.

Motivation for the material fatigue

- It is well known that plastic deformations lead to energy dissipation and material fatigue
- Material fatigue is manifested by material softening, heat release, material failure in finite time
- Very important: take into account the effects of energy exchange between heat and mechanical energy, thermal stresses, and material fatigue
- In particular great importance for the applications: methods for estimating the lifetime of oscillating thermoelastoplastic structures under material fatigue
- The existing mathematical literature mainly goes in the direction of the quasistatic apprach of elastoplastic processes with hysteresis; few results on the dynamics

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Literature concerning oscillating elastoplastic processes

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- Quasistatic approach

  W. Han, B.D. Reddy (1999); yield condition described by a sharp surface of plasticity - **M. Brokate, A.M. Khludnev** (2000); **M. Kuczma, P. Litewka, J. Rakoswki, J.R. Whiteman** (2004); **O. Millet, A. Cimetiere, A. Hamdouni** (2003)

- Γ-convergence approach

  (it doesn't extend to the case of oscillating systems) **D. Percivale** (1990); **G. Friesecke, R.D. James, S. Müller** (2006); **M. Liero, A. Mielke** (2010); **M. Liero, T. Roche** (preprint 2011) - They get the Prandtl-Ishlinskii model at the limit

- "Multiyield" character but without explicit reference to plates

  M. Brokate, C. Carstensen, J. Valdman (2004)

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The system of PDEs

- The resulting system from the theory developed by Krejčí and coworkers (starting point of our new project) is the following:

$$\partial_{tt} w - \partial_{tt} \Delta w + \mathbf{D}_2^* \sigma = g\,,$$

$$\sigma = \mathbf{B}\varepsilon + \int_0^\infty \mathfrak{s}_{rZ}[\varepsilon](t)\,\varphi(r)\,\mathrm{d}r$$

$$\varepsilon = \mathbf{D}_2 w$$

  where $\mathbf{D}_2$ is the second derivative operator $(\partial_{xx}, \partial_{yy}, \partial_{xy})$ and $\mathbf{D}_2^*$ is its adjoint
- We introduce a positive parameter $\theta > 0$ indicating the absolute temperature and $m(x,t) \geq 0$ a parameter which represents the material fatigue accumulated in the point x in the time interval $[0,t]$
- Basic modeling assumption: replacing the classical elastoplastic constitutive law with a new one, where we account for the material fatigue and where the memory keeps into consideration the temperature

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Coupling with the energy balance laws

- To the previous system we associate the **specific free energy**

$$\mathscr{F}[\theta,\varepsilon] = c_V\theta(1-\log(\theta/\theta_c)) + \frac{1}{2}\langle \mathbf{B}(m)\varepsilon,\varepsilon\rangle + \frac{1}{2}\int_0^\infty \langle \mathfrak{s}_{rZ}[\varepsilon],\mathfrak{s}_{rZ}[\varepsilon]\rangle \varphi(\theta,r)\,\mathrm{d}r - \beta(\theta-\theta_c)\langle\varepsilon,\mathbf{1}\rangle,$$

with a constant specific heat $c_V > 0$

- The **specific entropy** has the following form

$$\mathscr{S}[\theta,\varepsilon] = c_V\log(\theta/\theta_c) - \frac{1}{2}\int_0^\infty \langle \mathfrak{s}_{rZ}[\varepsilon],\mathfrak{s}_{rZ}[\varepsilon]\rangle \partial_\theta\varphi(\theta,r)\,\mathrm{d}r + \beta\langle\varepsilon,\mathbf{1}\rangle,$$

- from which, exploiting the well know relation $\mathscr{F} = \mathscr{U} - \theta\mathscr{S}$ we get the following form of the **internal energy**

$$\mathscr{U}[\theta,\varepsilon] = c_V\theta + \frac{1}{2}\langle \mathbf{B}(m)\varepsilon,\varepsilon\rangle + \frac{1}{2}\int_0^\infty \langle \mathfrak{s}_{rZ}[\varepsilon],\mathfrak{s}_{rZ}[\varepsilon]\rangle (\varphi(\theta,r)-\theta\partial_\theta\varphi(\theta,r))\,\mathrm{d}r + \beta\theta_c\langle\varepsilon,\mathbf{1}\rangle.$$

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$$\partial_t \mathscr{U}[\theta, \varepsilon] + \operatorname{div} \mathbf{q} = \langle \sigma, \partial_t \varepsilon \rangle \ ,$$

where $\mathbf{q}$ is the heat flux vector

- We derive the evolution law for the fatigue parameter which has to be compatible with the **Second Principle of Thermodynamics**, which we state in the form of the **Clausius-Duhem inequality**

$$\psi := \partial_t \mathscr{S}[\theta, \varepsilon] + \operatorname{div}\left(\frac{\mathbf{q}}{\theta}\right) \geq 0,$$

where ψ is the entropy production

- This implies that the **dissipation rate**

$$\begin{aligned} \mathscr{D} &= \langle \sigma, \partial_t \varepsilon \rangle - \partial_t \theta \mathscr{S}[\theta, \varepsilon] - \partial_t \mathscr{F}[\theta, \varepsilon] \\ &= -\frac{1}{2} \langle \mathbf{B}'(m)\varepsilon, \varepsilon \rangle \, \partial_t m + \int_0^\infty \langle \partial_t (\varepsilon - \mathfrak{s}_{rZ}[\varepsilon]), \mathfrak{s}_{rZ}[\varepsilon] \rangle \, \varphi(\theta, r) \, \mathrm{d}r \end{aligned}$$

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- The integral is non negative by virtue of the variational inequality which defines the stop operator ▸ Details
- The fatigue accumulation rate $\partial_t m$ should be nonnegative. Hence, it suffices to assume that $\mathbf{B}'(m)$ is a negative semidefinite matrix (softening!)
- The system will be complete by assuming the **linear Fourier law** between the heat flux and the temperature gradient

$$\mathbf{q} = -\kappa \nabla \theta\,,$$

- **Fundamental assumption:** proportionality between the rate of fatigue $\partial_t m$ and the dissipation rate $\mathscr{D}$

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The rainflow algorithm

- **Rainflow method for cyclic fatigue accumulation** - M. Endo (1968)
- The analysis of the rainflow algorithm in uniaxial processes has discovered a qualitative and quantitative relationship between accumulated fatigue and dissipated energy
- With an undamaged material, we associate fatigue value 0 , and 1 corresponds to total failure. Values between 0 and 1 quantify the degree of fatigue
- Experimental (decreasing!) curve $n(b)$ (the so-called Wöhler line) determines how many closed cycles of amplitude b lead to total failure
- With each closed cycle of amplitude b we associate the contribution $d(b) = \frac{1}{n(b)}$ of the individual cycle to total fatigue
- Therefore, the rainflow algorithm counts closed hysteresis loops in the loading history, and with each closed loop associates a number depending on its amplitude (the contribution of the loop to total damage) taken from the Wöler line

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- This corresponds to the mechanism of energy dissipation: the number associated with a closed loop is its area in this case
- Let a real loading process consist of a sequence of cycles with amplitudes b_j, $j = 1, \ldots, n$. The Palmgren-Miner additivity rule states that the total fatigue can be computed as the sum of all individual contributions $D_n = \sum_{j=1}^{n} d(b_j)$
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- For a numerical treatment of large data sets it is important to assume that oscillations of very small amplitude cause no damage
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- Let a real loading process consist of a sequence of cycles with amplitudes b_j, $j = 1, \ldots, n$. The Palmgren-Miner additivity rule states that the total fatigue can be computed as the sum of all individual contributions $D_n = \sum_{j=1}^{n} d(b_j)$
- When the contribution of a closed cycle to total fatigue is evaluated, the cycle is removed from the history. At the end of the process, the remaining *residual* of the input signal contains no more closed cycles
- For a numerical treatment of large data sets it is important to assume that oscillations of very small amplitude cause no damage
- The rainflow method is then *stable with respect to small measurement errors independently of the number of cycles*
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In multiassial processes?

- Drawback: the rainflow method is exclusively uniaxial - no closed cycles in the vector case
- In multiaxial loading processes, the concept of closed loop is meaningless, and no counterpart of the rainflow algorithm is known
- On the other hand, the notion of energy dissipation is independent of the experimental setting: experimental measurements at the point of material failure confirm strong temperature increase, which manifests an energy dissipation peak
- In fact, temperature tests are regularly used in engineering practice for damage analysis in high frequency regimes (e.g. in aircraft industry).

Fundamental assumption

Assuming that the fatigue rate is proportional to the dissipation rate, and that the material parameters depend on the fatigue parameter

- Assumption realistic: plastic deformations are driven by moving dislocations and ruptures of interatomic connections, which at the same time dissipate energy, and reduce the cohesion of the solid

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Evolution equation for the fatigue

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\begin{aligned}
\mathscr{D} &= \langle \sigma, \partial_t \varepsilon \rangle - \partial_t \theta \mathscr{S}[\theta, \varepsilon] - \partial_t \mathscr{F}[\theta, \varepsilon] \\
&= -\frac{1}{2} \left\langle \mathbf{B}'(m)\varepsilon, \varepsilon \right\rangle \partial_t m + \int_0^\infty \langle \partial_t(\varepsilon - \mathfrak{s}_{rZ}[\varepsilon]), \mathfrak{s}_{rZ}[\varepsilon] \rangle \, \varphi(\theta, r) \, \mathrm{d}r
\end{aligned}
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- The integral is non negative by virtue of the variational inequality which defines the stop operator
- The fatigue accumulation rate $\partial_t m$ should be nonnegative. Hence, it suffices to assume that $\mathbf{B}'(m)$ is a negative semidefinite matrix (softening!)
- **Fundamental assumption:** proportionality between the rate of fatigue $\partial_t m$ and the dissipation rate $\mathscr{D}$

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\left(\frac{1}{C(\theta)} + \frac{1}{2} \left\langle \mathbf{B}'(m)\varepsilon, \varepsilon \right\rangle \right) \partial_t m = \int_0^\infty \langle \partial_t(\varepsilon - \mathfrak{s}_{rZ}[\varepsilon]), \mathfrak{s}_{rZ}[\varepsilon] \rangle \, \varphi(\theta, r) \, \mathrm{d}r
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- **Motivation:**
  - possibility to account also for decreasing fatigue rate (in view of engineering applications)
  - the material can be partially repaired by local melting
- **How to achieve this goal:**
- Phase transition equation in the form of melting-solidification law

$$\alpha \chi_t \in -\partial_\chi \mathscr{F}[\varepsilon, \theta, \chi] \qquad \chi \in [0,1]$$

$\chi_0 \in [0,1]$ some initial condition, $A(x,t) := \int_0^t \frac{1}{\alpha}\left(\frac{L}{\theta_c}(\theta - \theta_c)\right)(x,\tau)\, d\tau$

$(\chi_t - A_t)(z - \chi) \geq 0$ for all $z \in [0,1]$

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$\boxed{\chi \in \mathfrak{s}_{[0,1]}[\chi_0, A]}$ $\qquad$ $\mathfrak{s}_{[0,1]}$ is a shifted stop

Thermodynamical consistency

- If we introduce $\mathscr{F}[\varepsilon,\theta,\chi]$ **specific free energy**, $\mathscr{S}[\varepsilon,\theta,\chi]$ **specific entropy** and $\mathscr{U}[\varepsilon,\theta,\chi]$ **internal energy** we are able to show that the first and second principles of thermodynamics are satisfied

$$\frac{\partial}{\partial t}\mathscr{U}[\varepsilon,\theta,\chi]+\mathrm{div}\,\mathbf{q}=\langle\sigma,\varepsilon_t\rangle \quad \text{(energy conservation)}$$

$$\frac{\partial}{\partial t}\mathscr{S}[\varepsilon,\theta,\chi]+\mathrm{div}\,\frac{\mathbf{q}}{\theta}\geq 0, \quad \text{(Clausius-Duhem inequality)}$$

- **Evolution equation for m:**

$$(C+\mathscr{K}[m,\theta,\varepsilon])\,m_t=-h(\chi_t)+\mathscr{D}[m,\theta,\varepsilon]$$

- where

$$\mathscr{D}[m,\theta,\varepsilon]:=\int_0^\infty \varphi(m,\theta,r)\langle \mathbf{K}\mathfrak{s}_{rZ}[\varepsilon],(\varepsilon-\mathfrak{s}_{rZ}[\varepsilon])_t\rangle\,\mathrm{d}r$$

$$\mathscr{K}[m,\theta,\varepsilon]:=-\frac{1}{2}\langle\mathbf{B}'(m)\varepsilon,\varepsilon\rangle-\frac{1}{2}\int_0^\infty \varphi_m(m,\theta,r)\langle\mathbf{K}\mathfrak{s}_{rZ}[\varepsilon],\mathfrak{s}_{rZ}[\varepsilon]\rangle\,\mathrm{d}r.$$

- allow the possibility of decreasing rate (i.e. $m_t<0$) but only in the case if χ grows faster than the plastic dissipation rate (strong melting)

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$$\frac{\partial}{\partial t}\mathscr{U}[\varepsilon,\theta,\chi]+\text{div}\mathbf{q}=\langle\sigma,\varepsilon_t\rangle \quad \text{(energy conservation)}$$

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- **Evolution equation for m:**

$$(C+\mathscr{K}[m,\theta,\varepsilon])\,m_t=-h(\chi_t)+\mathscr{D}[m,\theta,\varepsilon]$$

- where

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- Rainflow method for fatigue evaluation in elastoplastic materials (uniaxial cyclic loading) allow to consider dissipated energy as a measure for fatigue
- The solution cannot be expected to exist globally: singularities (thermal shocks) occur in finite time
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