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Local Stabilization of Fluid – Structure Models

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Outline of the talk

- Motivations and goals
- The PDE model, issues and method
- Rewriting the linearized/nonlinear model as a control system
- Stabilizability the linearized system - Stabilization of the nonlinear system
- Numerical experiments.

Motivations

- Stabilization of aerodynamic flows around an unstable stationary solution.

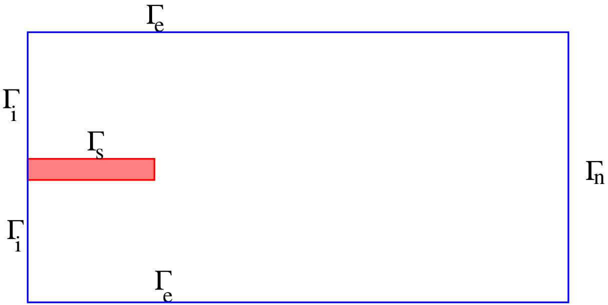
Given a fluid flow around an obstacle. Can we determine a control located at the boundary of the obstacle able to stabilize it in presence of perturbations.

- Autoregulation of either cerebrospinal flows or blood flows in the brain.

Can we model the autoregulation phenomenon by a feedback control corresponding to deformation of vessels ?

- In both case we have to deal with mixed boundary conditions. In both cases, we want to control the system by acting on an elastic structure located at the boundary of the fluid domain.

The geometrical configuration



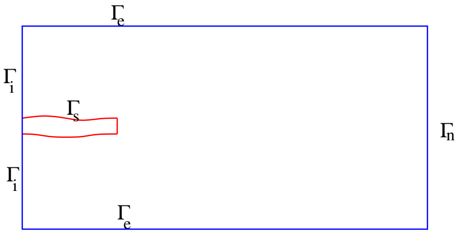
with Dirichlet-Neumann and Navier boundary conditions.

$$(\nu(\nabla u + \nabla u^T) - p\, I)n = \sigma(u,p)n = 0 \quad \text{on} \quad \Gamma_n \times (0,\infty),$$

$$u \cdot n = 0 \quad \text{and} \quad \varepsilon(u)n \cdot \tau = 0 \quad \text{on} \quad \Gamma_e \times (0,\infty).$$

For simplicity in writing we shall impose Neumann B.C. on Γ_e.

Due to the displacement $\eta(t)$ of the structure, the fluid equation is written in a time dependent geometrical domain



We introduce

$$\widetilde{Q}_\infty = \cup_{t\geq 0}\, \Omega_{\eta(t)} \times \{t\},$$

$$\widetilde{\Sigma}_s^\infty = \cup_{t\geq 0}\, \Gamma_{\eta(t)} \times \{t\}.$$

The model

The fluid equation

$$u_t + (u \cdot \nabla)u - \operatorname{div} \sigma(u,p) = 0, \quad \operatorname{div} u = 0 \quad \text{in } \widetilde{Q}_\infty,$$

$$u = \eta_2 \vec{e}_2 \text{ on } \widetilde{\Sigma}_s^\infty, \quad u = g_s + h \text{ on } \Sigma_i^\infty, \quad \sigma(u,p)n = 0 \quad \text{on } \Sigma_n^\infty$$

$$u(0) = u_0 \text{ in } \Omega_0,$$

$$\sigma(u,p) = \nu(\nabla u + \nabla u^T) - p\, I.$$

The structure equation

$$\eta_{1,t} = \eta_2 \quad \text{on } \Sigma_s^\infty,$$

$$\eta_{2,t} - \beta \Delta_s \eta_1 + \alpha \Delta_s^2 \eta_1 - \gamma \Delta_s \eta_2$$

$$= -\sigma(u,p)(-\eta_x \vec{e}_1 + \vec{e}_2) \cdot \vec{e}_2 - f_s + f\, \chi_{\Gamma_c} \quad \text{on } \Sigma_s^\infty,$$

$$\eta_1 = 0 \quad \text{and} \quad \frac{\partial \eta_1}{\partial n} = 0 \quad \text{on } \partial\Gamma_s \times (0,\infty),$$

$$\eta_1(0) = \eta_1^0 \quad \text{and} \quad \eta_2(0) = \eta_2^0 \quad \text{on } \Gamma_s.$$

The stationary solution

$$(u_s \cdot \nabla)u_s - \operatorname{div}\sigma(u_s, p_s) = 0, \quad \operatorname{div} u_s = 0 \quad \text{in } \Omega,$$

$$u_s = \eta_2 \vec{e}_2 \;\text{ on } \widetilde{\Gamma}_s, \quad u_s = g_s \;\text{ on } \Gamma_i, \quad \sigma(u_s, p_s)n = 0 \quad \text{on } \Gamma_n,$$

$$\eta_1 = \eta_2 \quad \text{on } \Gamma_s,$$

$$-\beta\Delta_s\eta_1 + \alpha\Delta_s^2\eta_1 - \gamma\Delta_s\eta_2 = -\sigma(u_s, p_s)(-\eta_x\vec{e}_1 + \vec{e}_2)\cdot\vec{e}_2 - f_s \quad \text{on } \Gamma_s,$$

$$\eta_1 = 0 \quad \text{and} \quad \frac{\partial\eta_1}{\partial n} = 0 \quad \text{on } \partial\Gamma_s \times (0, \infty).$$

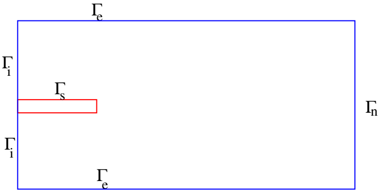
If $f_s = p_s$, then $\eta_1 = \eta_2 = 0$ and

$$(u_s \cdot \nabla)u_s - \operatorname{div}\sigma(u_s, p_s) = 0, \quad \operatorname{div} u_s = 0 \quad \text{in } \Omega,$$

$$u_s = 0 \;\text{ on } \Gamma_s, \quad u_s = g_s \;\text{ on } \Gamma_i, \quad \sigma(u_s, p_s)n = 0 \quad \text{on } \Gamma_n.$$

The change of variable is associated with the *structure displacement:*

Reference configuration $\xleftarrow{\mathcal{T}_{\eta_1}}$ Deformed configuration



The system in the reference configuration

$$
\begin{aligned}
&\hat{u}_t - \operatorname{div}\sigma(\hat{u},\hat{p}) + (\hat{u}\cdot\nabla)\hat{u} = \mathcal{F}[\hat{u},\hat{p},\eta_1,\eta_2],\\
&\operatorname{div}\hat{u} = \mathcal{G}[\hat{u},\eta_1] \text{ in } Q_\infty = \Omega\times(0,\infty),\\
&\hat{u} = \eta_2\,\vec{e}_2 \text{ on } \Sigma_s^\infty = \Gamma_s\times(0,\infty), \quad \hat{u} = g_s + h \text{ on } \Sigma_i^\infty = \Gamma_i\times(0,\infty),\\
&\sigma(\hat{u},\hat{p})n = 0 \text{ on } \Sigma_n^\infty = \Gamma_n\times(0,\infty), \quad \hat{u}(0) = \hat{u}^0 \text{ in } \Omega,\\
&\eta_{1,t} = \eta_2 \quad \text{ on } \Sigma_s^\infty,\\
&\eta_{2,t} - \beta\Delta_s\eta_1 + \alpha\Delta_s^2\eta_1 - \gamma\Delta_s\eta_2\\
&= \mathcal{H}[\hat{u},\hat{p},\eta_1] - p_s + f\,\chi_{\Gamma_c} \quad \text{ on } \Sigma_s^\infty,\\
&\eta_1 = 0 \quad\text{and}\quad \frac{\partial\eta_1}{\partial n} = 0 \quad \text{ on } \partial\Gamma_s\times(0,\infty),\\
&\eta_1(0) = \eta_1^0 \quad\text{and}\quad \eta_2(0) = \eta_2^0 \quad \text{ on } \Gamma_s.
\end{aligned}
$$

Classical approach

• Linearize the system around the unstable stationary solution.

Write the linearized PDE system as a control system on a Hilbert space Z.

• Study the stabilizability of the control system.

• Determine numerically a feedback for which we have a guarentee that if it is applied to the nonlinear PDE system, we still have a local stabilization result.

• For numerical issues. Determine a feedback stabilizing the linearized system projected onto a finite dimensional subspace $Z_u \subset Z$ containing

$$\oplus_{\mathrm{Re}(\lambda_i)\geq -\omega} G_{\mathbb{R}}(\lambda_i) \quad \text{for some } \omega > 0.$$

The linearized models

The linearized system (around 0) is

$$v_t - \operatorname{div} \sigma(v, q) = 0,$$

$$\operatorname{div} v = 0 \quad \text{in } Q_\infty,$$

$$v = \eta_2 \vec{e}_2 \ \text{ on } \Sigma_s^\infty, \quad v = h \ \text{ on } \Sigma_i^\infty, \quad \sigma(v, q)n = 0 \quad \text{on } \Sigma_n^\infty,$$

$$v(0) = v_0 \text{ in } \Omega,$$

$$\eta_{1,t} = \eta_2 \quad \text{on } \Sigma_s^\infty,$$

$$\eta_{2,t} - \beta\Delta_s\eta_1 + \alpha\Delta_s^2\eta_1 - \gamma\Delta_s\eta_2 = q + f\,\chi_{\Gamma_c} \quad \text{on } \Sigma_s^\infty,$$

$$\eta_1 = 0 \quad \text{and} \quad \frac{\partial\eta_1}{\partial n} = 0 \quad \text{on } \partial\Gamma_s \times (0, \infty),$$

$$\eta_1(0) = \eta_1^0 \quad \text{and} \quad \eta_2(0) = \eta_2^0 \quad \text{on } \Gamma_s.$$

The system linearized around $(u_s, p_s, 0, 0)$

$$
\begin{aligned}
& v_t - \operatorname{div}\sigma(v,q) + (u_s\cdot\nabla)v + (v\cdot\nabla)u_s - A_1\eta_1 - A_2\eta_2 = 0,\\
& \operatorname{div} v = A_3\eta_1 \quad \text{in } Q_\infty,\\
& v = \eta_2\, e_2 \quad \text{on } \Sigma_s^\infty, \quad v = h \quad \text{on } \Sigma_i^\infty,\\
& \sigma(v,q)n = 0 \quad \text{on } \Sigma_n^\infty, \quad v(0) = v^0 \text{ in } \Omega,\\
& \eta_{1,t} = \eta_2 \quad \text{on } \Sigma_s^\infty,\\
& \eta_{2,t} - \beta\Delta_s\eta_1 + \alpha\Delta_s^2\eta_1 - \gamma\Delta_s\eta_2 - A_4\eta_1 = q + f \quad \text{on } \Sigma_s^\infty,\\
& \eta_1 = 0 \quad \text{and} \quad \frac{\partial\eta_1}{\partial n} = 0 \quad \text{on } \partial\Gamma_s\times(0,\infty),\\
& \eta_1(0) = \eta_1^0 \quad \text{and} \quad \eta_2(0) = \eta_2^0 \quad \text{on } \Gamma_s.
\end{aligned}
$$

Elimination of the pressure with the Leray projector

$$V^0_{n,\Gamma_d}(\Omega) = \Big\{ v \in L^2(\Omega;\mathbb{R}^2) \mid \ \text{div } v = 0, \ v \cdot n = 0 \text{ on } \Gamma_d \Big\},$$

$$L^2(\Omega;\mathbb{R}^2) = V^0_{n,\Gamma_d}(\Omega) \oplus \text{grad } H^1_{\Gamma_n}(\Omega),$$

$$P \ : \ L^2(\Omega) \longmapsto V^0_{n,\Gamma_d}(\Omega).$$

The Stokes operator

$$D(A_0) = \Big\{ v \in V^1_{\Gamma_d}(\Omega) \mid$$

$$\exists p \in L^2(\Omega) \text{ s. t. } \mathrm{div}\sigma(v,p) \in L^2(\Omega;\mathbb{R}^2)$$

$$\text{and } \mathrm{div}\sigma(v,p)\, n = 0 \quad \text{on} \quad \Gamma_n \Big\},$$

$$A_0 v = P(\mathrm{div}\sigma(v,p)) \quad (\text{does not depend on } p).$$

The Oseen operator $(A, D(A))$ is defined by

$$D(A) = D(A_0) \quad \text{and} \quad Av = A_0 v + P\left((u_s\cdot\nabla)v + (v\cdot\nabla)u_s\right).$$

Due to the corners $\mathcal{J}_{d,n} \cup \mathcal{J}_{d,d}$, we have

$$D(A_0) \subset H^{3/2+\varepsilon_0}(\Omega;\mathbb{R}^2), \qquad \varepsilon_0 > 0.$$

Expression of the pressure

$$
\begin{aligned}
&-\Delta q = A_3\eta_{1,t} + \mathrm{div}((u_s\cdot\nabla)v + (v\cdot\nabla)u_s) - \mathrm{div}(A_1\eta_1) - \mathrm{div}(A_2\eta_2) \quad \mathrm{i}\\
&q = 2\nu\,\varepsilon(v)n\cdot n \qquad \text{on } \Sigma_n^\infty,\\
&\frac{\partial q}{\partial n} = 2\nu\,\mathrm{div}\varepsilon(v)\cdot n - v_t\cdot n = 2\nu\,\mathrm{div}\varepsilon(v)\cdot n - \eta_{2,t}.
\end{aligned}
$$

Thus

$$q = -N_s(\eta_{2,t}) + N_d(A_3\,\eta_{1,t}) + N_v(v) + N(A_1\eta_1) + N(A_2\eta_2).$$

Paradox.
$$
\begin{aligned}
&N_s(\eta_{2,t}) + N_d(A_3\,\eta_{1,t}) \in L^2_{loc}([0,\infty); H^{3/2+\varepsilon_0}(\Omega)),\\
&N(A_1\eta_1) + N(A_2\eta_2) \in L^2_{loc}([0,\infty); H^1(\Omega)),\\
&N_v(v) \in L^2_{loc}([0,\infty); L^2(\Omega)),
\end{aligned}
$$

and

$$q \in L^2_{loc}([0,\infty); H^{1/2+\varepsilon_0}(\Omega)).$$

The paradox can be solved for the resolvent equation.

Equivalent formulation of the PDE system

$$M_a z' = \widehat{\mathcal{A}} z + \mathcal{B} h, \quad z(0) = z_0,$$

$$(I - P)v(t) = (I - P)L(\eta_2(t)\, e_2, A_3\eta_1(t)),$$

$$z = (Pv, \eta_1, \eta_2)^T, \quad \mathcal{B} = (0\ 0\ I_{L^2(\Gamma_s)})^T,$$

L is the lifting operator of the divergence and Dirichlet boundary condition, and

$$\widehat{\mathcal{A}} = \begin{pmatrix} A & (PA_1 - APL(0, A_3)) & (PA_2 - APL(\cdot, 0)) \\ 0 & 0 & I \\ \gamma_s N_s & A_{\alpha,\beta} + \cdots & \gamma\Delta + \cdots \end{pmatrix},$$

and

$$M_a = \begin{pmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & \gamma_s N_d A_3 & I + \gamma_s N_s \end{pmatrix}.$$

The added mass operator, M_a, is no longer symmetric.

We set

$$\mathcal{A} = M_a^{-1}\,\widehat{\mathcal{A}},$$

and, due to the corners

$$D(\mathcal{A}) =$$

$$\Big\{(Pv,\eta_1,\eta_2) \in H^{1/2+\varepsilon_0}(\Omega;\mathbb{R}^2) \times (H^4 \cap H_0^2)(\Gamma_s) \times H_0^2(\Gamma_s)$$

$$\mid Pv - PL(\eta_2\,\vec{e}_2, A_3\eta_1) \in D(A_0)\Big\}.$$

Theorem. The operator $(\mathcal{A}, D(\mathcal{A}))$ is the infinitesimal generator of an analytic semigroup on $Z = V^0_{n,\Gamma_d}(\Omega) \times H^2_0(\Gamma_s) \times L^2(\Gamma_s)$.

Analyticity of the Oseen operator + Analyticity for the structure (Chen-Triggiani) + perturbation arguments

Theorem. The resolvent of $(\mathcal{A}, D(\mathcal{A}))$ is compact in Z.

Paradox. For the solution to

$$z' = \mathcal{A}z + F, \quad z(0) = z_0, \quad F \quad \text{and} \quad z_0 \text{ regular}$$

we have

$$(Pv, \eta_1, \eta_2) \in L^2_{loc}([0,\infty); D(\mathcal{A})) \cap H^1_{loc}([0,\infty); Z),$$

$$Pv \in L^2_{loc}([0,\infty); H^{1/2+\varepsilon_0}(\Omega; \mathbb{R}^2)).$$

$$\text{But} \qquad v \in L^2_{loc}([0,\infty); H^{3/2+\varepsilon_0}(\Omega; \mathbb{R}^2)).$$

Trick. Both Pv and $(I-P)v$ belongs to $L^2_{loc}([0,\infty); L^2(\Omega; \mathbb{R}^2))$.

Regularity results in the presence of mixed B.C.

If $v_0 \in H^1_{\Gamma_i}(\Omega;\mathbb{R}^2)$, $\mathrm{div} v_0 = 0$, $\eta_1^0 \in H^3(\Gamma_s) \cap H^2_0(\Gamma_s)$, $\eta_2^0 \in H^1_0(\Gamma_s)$, $v_0 = \eta_2^0 \vec{e}_2$ on Γ_s, the solution to the linearized system belongs to

$$
\begin{aligned}
& v \in L^2_{loc}([0,\infty); H^2_\delta(\Omega;\mathbb{R}^2)) \cap H^1(0,\infty; L^2(\Omega;\mathbb{R}^2)) = H^{2,1}_\delta(Q_\infty;\mathbb{R}^2),\\
& p \in L^2_{loc}([0,\infty); H^1_\delta(\Omega;\mathbb{R}^2)),\\
& \eta_1 \in L^2_{loc}([0,\infty); H^4(\Gamma_s)) \cap H^2(0,\infty; L^2(\Gamma_s)) = H^{4,2}(\Sigma_s^\infty),\\
& \eta_2 \in L^2_{loc}([0,\infty); H^2(\Gamma_s)) \cap H^1(0,\infty; L^2(\Gamma_s)) = H^{2,1}(\Sigma_s^\infty).
\end{aligned}
$$

$$
\begin{aligned}
\|v\|^2_{H^2_\delta(\Omega;\mathbb{R}^2)} &= \sum_{|k|=0}^{2} \sum_{i=1}^{2} \int_\Omega \prod_{j\in \mathcal{J}_{d,n}\cup\mathcal{J}_{d,d}} r_j^{2\delta} |\partial_k v|^2 dx,\\
\|p\|^2_{H^1_\delta(\Omega)} &= \sum_{|k|=0}^{1} \sum_{i=1}^{2} \int_\Omega \prod_{j\in \mathcal{J}_{d,n}\cup\mathcal{J}_{d,d}} r_j^{2\delta} |\partial_k p|^2 dx,
\end{aligned}
$$

where r_j stands for the distance to the junction point $J_j \in \mathcal{J}_{d,n} \cup \mathcal{J}_{d,d}$.

Due to the right angles at the Neumann-Dirichlet junctions and Dirichlet-Dirichlet junctions, we have

$$
\begin{aligned}
&H^2_\delta(\Omega;\mathbb{R}^2) \subset H^{3/2+\varepsilon_0}(\Omega;\mathbb{R}^2),\\
&H^1_\delta(\Omega) \subset H^{1/2+\varepsilon_0}(\Omega;\mathbb{R}^2), \quad \text{for some } \varepsilon_0 > 0.
\end{aligned}
$$

Control strategy

- Choose

$$Z_u = \oplus_{j \in J_u} G_{\mathbb{R}}(\lambda_j) \quad \text{with} \quad Z = Z_u \oplus Z_s$$

and

$$Z_u^* = \oplus_{j \in J_u} G_{\mathbb{R}}^*(\lambda_j) \quad \text{with} \quad Z^* = Z = Z_u^* \oplus Z_s^*.$$

- Determine a basis $\{e_1, \cdots, e_{d_u}\}$ of Z_u and a basis $\{\Phi_1, \cdots, \Phi_{d_u}\}$ of Z_u^* satisfying

$$(e_i, \Phi_j)_Z = \delta_{i,j}.$$

- These bases are used to determine the projected linearized system and therefore the feedback operator.

The eigenvalue problem for $\mathcal{A}$

$$\lambda v - \operatorname{div}\sigma(v,q) + (u_s\cdot\nabla)v + (v\cdot\nabla)u_s - A_1\eta_1 - A_2\eta_2 = 0,$$

$$\operatorname{div} v = A_3\eta_1 \quad \text{in } \Omega,$$

$$v = \eta_2\, e_2 \quad \text{on } \Gamma_s, \quad v = 0 \quad \text{on } \Gamma_i,$$

$$\sigma(v,q)n = 0 \quad \text{on } \Gamma_n,$$

$$\lambda\eta_1 = \eta_2 \quad \text{on } \Gamma_s,$$

$$\lambda\eta_2 - \beta\Delta_s\eta_1 + \alpha\Delta_s^2\eta_1 - \gamma\Delta_s\eta_2 - A_4\eta_1 = q \quad \text{on } \Gamma_s,$$

$$\eta_1 = 0 \quad \text{and} \quad \frac{\partial\eta_1}{\partial n} = 0 \quad \text{on } \partial\Gamma_s.$$

The eigenvalue problem for $\mathcal{A}^*$

$$\lambda\phi - \operatorname{div}\sigma(\phi,\psi) - (u_s \cdot \nabla)\phi + (\nabla u_s)^T\phi = 0,$$

$$\operatorname{div}\phi = 0 \quad \text{in } \Omega,$$

$$\phi = \zeta_2 \vec{e}_2 \ \text{ on } \ \Gamma_s, \quad \phi = 0 \ \text{ on } \ \Gamma_0,$$

$$\sigma(\phi,\psi)n + u_s \cdot n\,\phi = 0 \ \text{ on } \ \Gamma_n,$$

$$\lambda\zeta_1 + (-A_{\alpha\beta})^{-1}(\zeta_2 + A_3^*\psi - A_1^*\phi - A_4^*\zeta_2) = 0 \quad \text{in } \Gamma_s,$$

$$\lambda\zeta_2 + \beta\Delta_s\zeta_1 - \delta\Delta_s\zeta_2 - \alpha\Delta_s^2\zeta_1 - A_2^*\phi = \psi \ \text{ in } \Gamma_s,$$

$$\zeta_1 = 0 \quad \text{and} \quad \frac{\partial\zeta_1}{\partial n} = 0 \quad \text{on } \partial\Gamma_s.$$

• What is the relationship between the PDE eigenvalue problem for the direct system and the eigenvalue problem for $\mathcal{A}$?

• What is the relationship between the PDE eigenvalue problem for the adjoint system and the eigenvalue problem for $\mathcal{A}^*$?

• What is the relationship between the bi-orthogonality conditions for based on eigenfunctions for $\mathcal{A}$ and $\mathcal{A}^*$ and bi-orthogonality conditions for based on eigenfunctions for the direct and adjoint PDE systems ?

$M_a^*(P\phi, \zeta_1, \zeta_2)$ is an eigenvector for $\mathcal{A}^*$ associated with λ and $(I - P)\phi = (I - P)L(\zeta_2\vec{e}_2, 0)$ iff (ϕ, ζ_1, ζ_2) is an eigenvector for the adjoint PDE system associated with λ.

Equivalence of bi-orthogonality conditions

The bi-orthogonality condition for eigenfunctions of the PDE systems is equivalent to the bi-orthogonality condition for eigenfunctions of $\mathcal{A}$ and $\mathcal{A}^*$.

$$((v_i, \eta_{1,i}, \eta_{2,i}), (\phi_j, \zeta_{1,j}, \zeta_{2,j}))_{L^2} = \delta_{i,j}$$

is equivalent to

$$((Pv_i, \eta_{1,i}, \eta_{2,i}), M_a^*(P\phi_j, \zeta_{1,j}, \zeta_{2,j}))_Z = \delta_{i,j}$$

Stabilizability of the linearized F–S system

We have to check the following unique continuation property.

If $(\lambda, \phi, \psi, \zeta_1, \zeta_2)$ is solution to the following eigenvalue problem

$$\lambda\phi - \operatorname{div}\sigma(\phi,\psi) - (u_s\cdot\nabla)\phi + (\nabla u_s)^T\phi = 0 \quad \text{and} \quad \operatorname{div}\phi = 0 \quad \text{in } \Omega,$$
$$\phi = \zeta_2\vec{e}_2 \ \text{ on } \ \Gamma_s, \quad \phi = 0 \ \text{ on } \ \Gamma_i, \quad \sigma(\phi,\psi)n + u_s\cdot n\,\phi = 0 \ \text{ on } \ \Gamma_n,$$
$$\lambda\zeta_1 + \zeta_2 + A_1^*\phi + A_3^*\psi + B_1^*\zeta_2 = 0 \quad \text{in } \Gamma_s,$$
$$\lambda\zeta_2 + \beta\Delta_s\zeta_1 - \delta\Delta_s\zeta_2 - \alpha\Delta_s^2\zeta_1 - A_2^*\phi = \psi \ \text{ in } \Gamma_s,$$
$$\zeta_1 = 0 \quad \text{and} \quad \frac{\partial\zeta_1}{\partial n} = 0 \quad \text{on } \partial\Gamma_s.$$

with $\operatorname{Re}\lambda \geq -\omega$ and

$$B^*(P\phi,\zeta_1,\zeta_2) = \zeta_2\,\chi_{\Gamma_c} = 0,$$

then

$$\phi = 0, \quad \psi = 0, \quad \zeta_1 = \zeta_2 = 0.$$

• We assume that the spectrum of the linearized Stokes operator is disjoint to the spectrum of the damped beam operator.

• We can choose a control space of finite dimension so that the unique continuation property reduces to the unique continuation property for the Oseen oeprator.

• Under that condition and if $u_s = 0$, we show that there exists a control space of finite dimension for which the system is stabilizable.

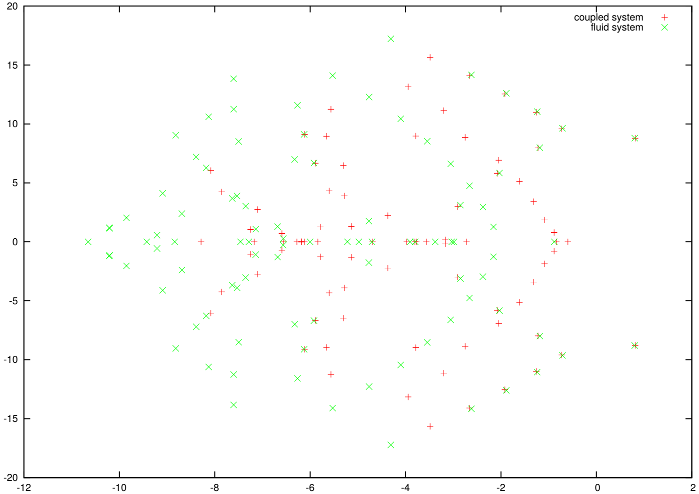
• If $u_s \neq 0$ and if u_s is regular enough, the technique by Osses and Puel for the Stokes equation (09) may seem to be adapted, but it is not yet done.

Local stabilization of the closed loop nonlinear F–S system

If $u_0 \in H^1_{\Gamma_i}(\Omega;\mathbb{R}^2)$, $\eta_1^0 \in H^3(\Gamma_s) \cap H^2_0(\Gamma_s)$, $\eta_2^0 \in H^1_0(\Gamma_s)$, $u_0 = \eta_2^0\,\vec{e}_2$ on Γ_s, and $\mathrm{div} u_0 = \mathcal{G}(u_0, \eta_1)$, and if $(u_0 - u_s, p_0, p_s, \eta_1^0, \eta_2^0)$ is small enough in $H^1_{\Gamma_i}(\Omega;\mathbb{R}^2) \times H^3(\Gamma_s) \cap H^2_0(\Gamma_s) \times H^1_0(\Gamma_s)$, then the closed loop nonlinear system admits a solution decaying exponentially to the stationary solution in $H^{2,1}_\delta(Q_\infty;\mathbb{R}^2) \times H^{4,2}(\Sigma_s^\infty) \times H^{2,1}(\Sigma_s^\infty)$.

Numerical results

The spectrum of the controlled system – $\mathrm{Re} = \frac{U_m\, e}{\nu} = 200.$



Method.

- We project the linearized system onto the sum of the generalized eigenspace generated by the 2 unstable eigenvalues. It is of dimension 2 and the control is of dimension 4 (2 controls on each beam).
- We determine a feedback stabilizing the projected system.
- The feedback law is next applied to the full nonlinear system.

Stabilizability issues.

- The boundary control is chosen of the form

$$f(x,t) = \sum_{i=1}^{N_c} f_i(t)\,\xi_i(x).$$

The functions $(\xi_i)_{1\leq i\leq N_c}$ can be chosen to prove that the linearized system around the null solution is stabilizable.

- The functions $(\xi_i)_{1\leq i\leq N_c}$ can be chosen to prove that the linearized system around an unstable stationary solution is stabilizable (under investigation – verified by numerical calculations).

The projected system

We denote by

- $\mathbf{E}_u \in \mathbb{R}^{N_z \times d_u}$ the matrix whose columns are $(e^1, \cdots, e^{d_u})$,
- $\mathbf{\Xi}_u$ the matrix whose columns are $(\boldsymbol{\xi}^1, \cdots, \boldsymbol{\xi}^{d_u})$.

We set

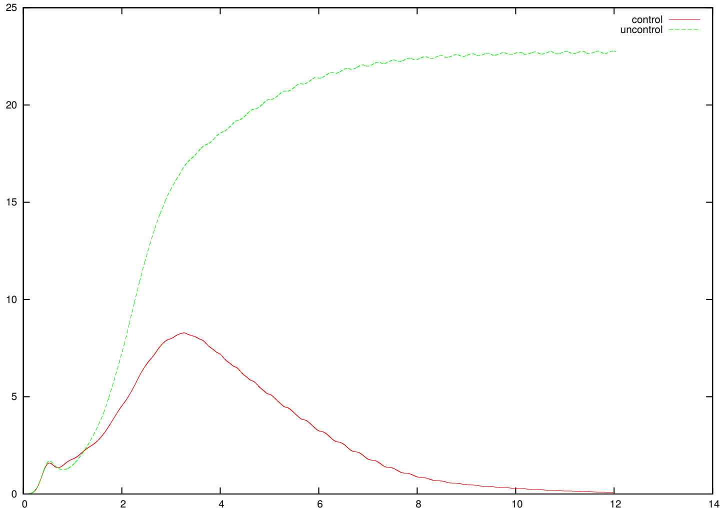
$$\Lambda_u = \mathbf{\Xi}_u^T A_{zz} \mathbf{E}_u \quad \text{and} \quad B_u = \mathbf{\Xi}_u^T B.$$

The projected system is

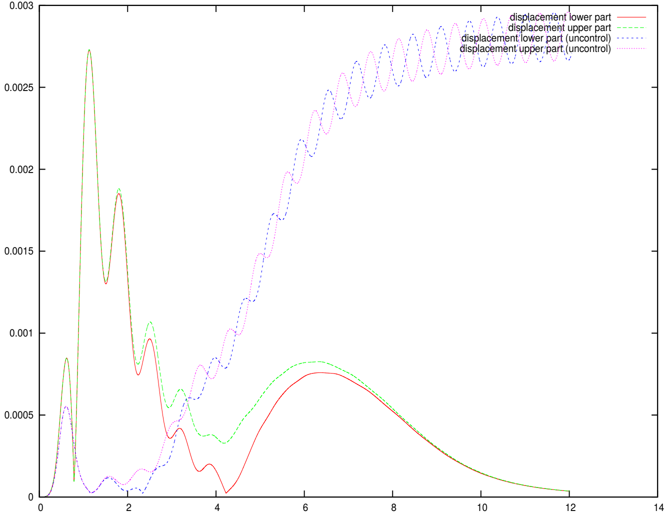
$$\zeta' = \Lambda_u \zeta + B_u f, \quad \zeta(0) = \zeta_0.$$

It is used to determine the feedback which next applied to the semi-discrete PDE system.

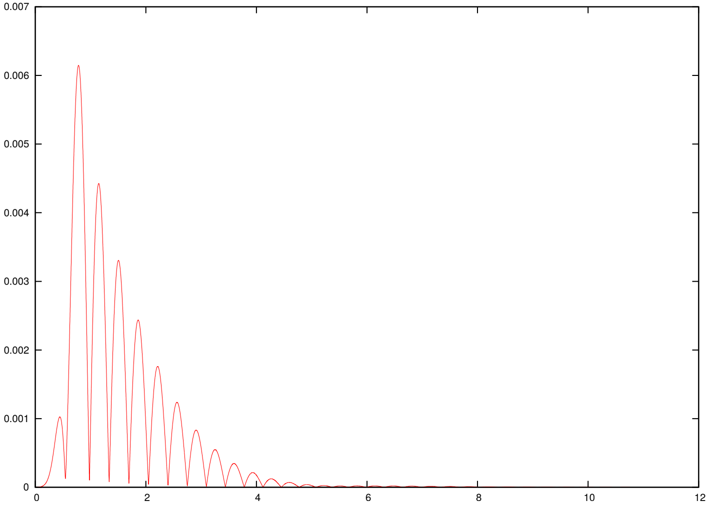
Numerical results for the nonlinear system – Velocity norm



Displacement



Control norm



We have obtained similar stabilization results for the system coupling the NSE and the Lamé system, with a control acting only in the Lamé system R-V, in preparation.

Conclusion.

- We are able to stabilize a fluid flow over and below a thick plate around an unstable stationary solution at a moderate Reynolds number $Re = 200$.

- The stabilization is done by boundary deformations by using only the two unstable eigenmodes.

- Numerical issues. Use localized deformation and more than two unstable fluid eigenmodes to determine the feedback control.

- Stabilizability issues when $u_s \neq 0$ are difficult.

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Thank you for your attention