Boundary stabilization of parabolic equations

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Settings of the problem

Let $\mathcal{O} \subset \mathbb{R}^d$, $d \in \mathbb{N}^*$, be a bounded and open domain with smooth boundary $\partial\mathcal{O} = \Gamma_1 \cup \Gamma_2$.

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With $f, f_p \in C(\overline{\mathcal{O}} \times \mathbb{R})$;
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with constants $C, \mu > 0$ and p_o in some set.

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Its eigenvalues $\{\lambda_i\}_{i=1}^{\infty}$ and its corresponding eigenfunctions $\{\phi_i\}_{i=1}^{\infty}$.
There exists $N \in \mathbb{N}$ such that

$$\lambda_i \geq 0 \text{ for } i = 1, ..., N \text{ and } \lambda_i < 0 \text{ for } i \geq N+1.$$

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V. Barbu showed that the feedback

$$u = \eta \left\langle \mathbf{B}^{-1} \begin{pmatrix} \langle y, \phi_1 \rangle \\ \langle y, \phi_2 \rangle \\ .... \\ \langle y, \phi_N \rangle \end{pmatrix}, \begin{pmatrix} \mu_1 \frac{\partial}{\partial \mathbf{n}} \phi_1 \\ \mu_2 \frac{\partial}{\partial \mathbf{n}} \phi_2 \\ ... \\ \mu_N \frac{\partial}{\partial \mathbf{n}} \phi_N \end{pmatrix} \right\rangle_N$$

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assures the stability provided that the system $\left\{\frac{\partial}{\partial \mathbf{n}}\phi_i\right\}_{i=1}^N$ is linearly independent.

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$$\frac{\partial}{\partial t}\langle y, \phi_j\rangle + \frac{-(k-\lambda_j)\lambda_j + k\eta}{k-\lambda_j-\eta}\langle y, \phi_j\rangle = 0,\ j = 1, ..., N.$$

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So, for η large

$$\frac{-(k-\lambda_j)\lambda_j+k\eta}{k-\lambda_j-\eta}>0.$$

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Simple eigenvalues:

$$0 \leq \lambda_N < \lambda_{N-1} < ... < \lambda_1.$$

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Let the matrices

$$\Lambda_{\gamma_k} := \begin{pmatrix} \frac{1}{\gamma_k - \lambda_1} & 0 & ... & 0 \\ 0 & \frac{1}{\gamma_k - \lambda_2} & ... & 0 \\ ... & ... & ... & ... \\ 0 & 0 & ... & \frac{1}{\gamma_k - \lambda_N} \end{pmatrix}, \ k = 1, ..., N, \tag{4}$$

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$$\Lambda_S = \sum_{k=1}^{N} \Lambda_{\gamma_k}.$$

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Then, the stabilizing feedback is

$$u = \left\langle \Lambda_S A \begin{pmatrix} \langle y, \phi_1 \rangle \\ \langle y, \phi_2 \rangle \\ ... \\ \langle y, \phi_N \rangle \end{pmatrix}, \begin{pmatrix} \frac{\partial}{\partial \nu} \phi_1(x) \\ \frac{\partial}{\partial \nu} \phi_2(x) \\ ... \\ \frac{\partial}{\partial \nu} \phi_N(x) \end{pmatrix} \right\rangle_N .$$

The non-singularity of the sum $B_1 + ... + B_N$.

Assume that there is $z = \begin{pmatrix} z_1 \\ z_2 \\ ... \\ z_N \end{pmatrix} \in \mathbb{R}^N$, nonzero, such that

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It yields the homogeneous system

$$\sum_{i=1}^{N} z_i \frac{1}{\gamma_k - \lambda_i} \frac{\partial}{\partial \nu} \phi_i(x) = 0, \ \forall 1 \leq k \leq N.$$

The non-singularity of the sum $B_1 + ... + B_N$.

The determinant of the matrix of the system is

$$\begin{vmatrix} \frac{1}{\gamma_1-\lambda_1}\frac{\partial}{\partial\nu}\phi_1(x) & \frac{1}{\gamma_1-\lambda_2}\frac{\partial}{\partial\nu}\phi_2(x) & ... & \frac{1}{\gamma_1-\lambda_N}\frac{\partial}{\partial\nu}\phi_N(x) \\ \frac{1}{\gamma_2-\lambda_1}\frac{\partial}{\partial\nu}\phi_1(x) & \frac{1}{\gamma_2-\lambda_2}\frac{\partial}{\partial\nu}\phi_2(x) & ... & \frac{1}{\gamma_2-\lambda_N}\frac{\partial}{\partial\nu}\phi_N(x) \\ ... & ... & ... & ... \\ \frac{1}{\gamma_N-\lambda_1}\frac{\partial}{\partial\nu}\phi_1(x) & \frac{1}{\gamma_N-\lambda_2}\frac{\partial}{\partial\nu}\phi_2(x) & ... & \frac{1}{\gamma_N-\lambda_N}\frac{\partial}{\partial\nu}\phi_N(x) \end{vmatrix}$$

$$= \prod_{i=1}^{N}\frac{\partial}{\partial\nu}\phi_i(x)\begin{vmatrix} \frac{1}{\gamma_1-\lambda_1} & \frac{1}{\gamma_1-\lambda_2} & ... & \frac{1}{\gamma_1-\lambda_N} \\ \frac{1}{\gamma_2-\lambda_1} & \frac{1}{\gamma_2-\lambda_2} & ... & \frac{1}{\gamma_2-\lambda_N} \\ ... & ... & ... & ... \\ \frac{1}{\gamma_N-\lambda_1} & \frac{1}{\gamma_N-\lambda_2} & ... & \frac{1}{\gamma_N-\lambda_N} \end{vmatrix} \neq 0, \text{ a.e. } x \in \Gamma_1,$$

Sketch of the proof

$$\mathcal{Z}_t = -\gamma_1 \mathcal{Z} + \sum_{k=2}^{N} (\gamma_1 - \gamma_k) B_k A \mathcal{Z}, \ \ t > 0; \ \ \mathcal{Z}(0) = \mathcal{Z}_o. \tag{5}$$

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$$\frac{1}{2}\frac{d}{dt}\|A^{\frac{1}{2}}\mathcal{Z}(t)\|_N^2 = -\gamma_1\|A^{\frac{1}{2}}\mathcal{Z}(t)\|_N^2 + \sum_{k=2}^{N}(\gamma_1 - \gamma_k)\left\langle B_k A\mathcal{Z}(t), A\mathcal{Z}(t)\right\rangle_N, \tag{6}$$

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To appear in International Journal of Control

Applications

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