Mathematical Theory of PDE Dynamics Arising in Flow Structure Interactions.

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Acknowledgments

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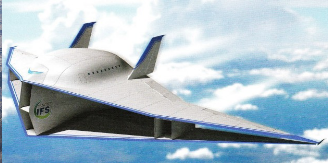
- Earl Dowell, Duke University.
- Justin Webster, NC-State.

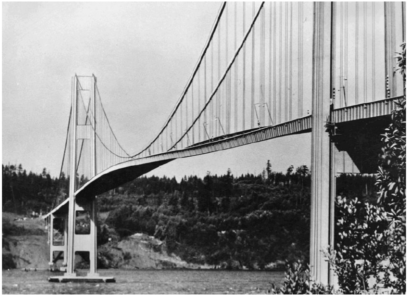
Flow Structure Interactions- Applications

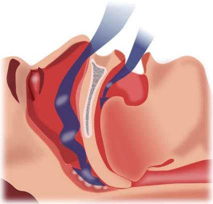
- **Aerodynamics.** Control of **flutter**. Flutter speed.
  - Subsonic, supersonic and transonic regimes;
- **Large space structures.** Large and thin. Highly oscillatory.
- **Medical Sciences**
  - Treatment of apnea
- **Engineering**
  - Bridges and Buildings
  - Windmills

Applications -flow structure supersonic



Applications-flow structure subsonic





Outline

- **Physical goals** -motivation.
- Formulate the problem within **mathematical setting** -identify mathematical problem to study.
- **PDE Models**- Nonlinear Dynamics **Hyperbolic /Hyperbolic-like** with **interface.**
- How to formulate a physical problem within the context of PDE's: **Role of Modeling** supported by Numerics and Experiment.
- **Main Results**
  - **Representation as a wellposed Dynamical System** (S_t, X)**.**
  - **Stability and long time behavior**
  - **Global attractors.**
  - **Control of the dynamics**
- Ideas about Proofs
- Conclusions and **Open Problems**

Goals and Challenges

- Eliminate the flutter, if possible.
- or control the flutter speed? Where? On the structure.

The resulting PDE system has

- No Dissipation
- No Compactness
- Degeneracy of Ellipticity in the Energy Function.[Supersonic]

Question: How to formulate a mathematical question pertinent to these physical problems?

Answer: Given PDE model: study the following:

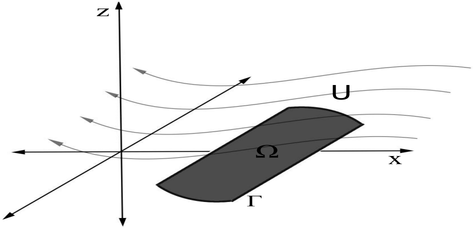
- **Generation** of a nonlinear evolution. Existence of dynamical system (S_t, X).
- Strong **stability** to equilibria
- Uniform **attraction** of evolution to a **finite dimensional set.**
- **Control** the resulting finite dimensional dynamical system.

PDE model

- **PDE Model-Flow / Structure Interaction.**
  - E.H. Dowell **Aeorelasticity of plates and shells**, Nordhof 1975, 2004
  - V. Bolotin, *Nonconservative Problems of Elasticity* , Pergamon,1963
  - J.D Cole, L.P. Cook, *Transonic aerodynamics, North Holland, 1986*
  - A.V. Balakrishnan, *Aeroelasticity; Continuum Theory* , Springer Verlag, 2012.
- **Experimental studies** -wind tunnel:
  NASA Lab at UCLA ,
  AFOSR/NASA Workshop at UCLA 2011,
  Earl H. Dowell and his group, Duke Univ.
- **Numerical studies:** E. Dowell and his group at Duke.

PDE Model.

- Thin, flexible plate, **moving with a velocity** U ($U = 1$ normalised speed of sound).
- Ω is a closed, two-dimensional domain (smooth) in the x-y plane.
- The unperturbed flow is in the negative x-direction.



$\phi(x, y, z; t)$ - **velocity potential at a point,** $u(x, y, t)$ **vertical displacement.**

Structure

Structural equation-$u(t,x,y)$

$$u_{tt} + \Delta^2 u + k(u_t) + \mathbf{f}(\mathbf{u}) = p_0 + (\partial_t + U\partial_x)\phi\big|_\Omega \ \ in \ \Omega$$
$$Boundary \ \ conditions: BC(u) = Clamped \ \ on \ \partial\Omega$$

Initial conditions: $u(0) = u_0 \in H_0^2(\Omega), u_t(0) = u_1 \in L_2(\Omega)$

$k(u_t)$ - Feedback control placed on the structure,

$p_0 \in L_2(\Omega)$ is the static aerodynamic pressure on the plate surface.

$f(u)$ is the nonlinearity -internal force $f(u) = -[\mathcal{F}(u), u]$, $\mathcal{F}(u)$ Airy's stress.

aeroelastic potential $= tr_U(\phi) = (\partial_t + U\partial_x)\phi|_\Omega$,

Flow

Flow equation -$\phi(t,x,y,z)$

$$
\begin{aligned}
(\partial_t + U\partial_x)^2\phi = \Delta\phi - \partial_x\phi\partial_x^2\phi \;\; in \;\; R^3_+ \\
Boundary\;\; conditions: \partial_\nu\phi\Big|_{z=0} = -(\partial_t + U\partial_x)u(\mathbf{1}_\Omega(x,y)) \;\; on \;\; \Omega \\
Initial\;\; conditions: \;\; \phi(t=0)=\phi_0,\;\; \phi_t(t=0)=\phi_1
\end{aligned}
$$

- aeroelastic potential $= tr_U(\phi) = (\partial_t + U\partial_x)\phi|_\Omega$,
- downwash $= tr_U(u) = (\partial_t + U\partial_x)u$

PDE -system

A nonlinear plate and perturbed wave, coupled at the interface $\Omega \subset \mathbb{R}^2$:

$$\begin{cases} u_{tt} + \Delta^2 u + k(u_t) + \mathbf{f}(\mathbf{u}) = p_0 + tr_U(\phi)|_\Omega \ \ in\ \Omega \\ BC(u) = Clamped\ \ on\ \partial\Omega \\ (\partial_t + U\partial_x)^2\phi = \Delta\phi - \partial_x\phi\partial_x^2\phi \ \ in\ R^3_+ \\ \partial_\nu\phi\big|_{z=0} = -tr_U(u)(\mathbf{1}_\Omega(x,y)) \ \ on\ \Omega \\ u(t=0) = u_0,\ u_t(t=0) = u_1;\ \phi(t=0) = \phi_0,\ \phi_t(t=0) = \phi_1 \end{cases}$$

aeroelastic potential, downwash

$f(u) = -[\mathcal{F}(u), u]$.
Two **hyperbolic -like** dynamics coupled on the interface Ω.

Von Karman -Airy Stress function nonlinearity

- $f(u) = -[u, \mathcal{F}(u) - F_0]$. F_0 is in-plane loading .
- $[g, h] = g_{xx}h_{yy} + g_{yy}h_{xx} - 2g_{xy}h_{xy}$ is the von Karman bracket.
- $\mathcal{F}(u)$ is the Airy Stress function, solves

$$\begin{cases} \Delta^2 v = -[u, u] & \text{in } \Omega \\ \mathcal{F}(u) = \frac{\partial \mathcal{F}(u)}{\partial \nu} = 0 & \text{on } \Gamma \end{cases}$$

$f(u)$ cubic and *nonlocal* $f : H^2(\Omega) \to H^{-\epsilon}(\Omega)$, $\epsilon > 0$, J.L.Lions

Goals:

Wellposedness of finite energy solutions

Long Time Behavior

Expectations: based on experimental studies:

- Elimination of the **flutter at the subsonic level** $U < 1$
- Asymptotic **reduction of structural dynamics to a finite dimensional attracting sets** (chaotic).

Mathematical challenges:

- lack of active dissipation on the flow and the structure;
- lack of dissipativity,compactness/regularity;
- potential degeneracy of the energy function.

Work done in the past

- Wellposedness of **finite energy** solutions:
  - **Regularized Models**- containing inertial-rotational forces, $\gamma \Delta u_{tt}$ added. Implies $u_t \in C(H^1(\Omega))$
  - **Strongly damped** plate equation: **regularizing effect is parabolic** : Δu_t added to the model. Implies $u_t \in L_2(H^1(\Omega))$
  - **Thermal effects** included, **making plate eq. analytic semigroup.**

In all these cases $u_t \in H^1(\Omega)$, RATHER than $u_t \in L_2(\Omega)$.
$u_t \in L_2 \Rightarrow f(u)$ supercritical; $u_t \in H^1 \Rightarrow f(u)$ subcritical, compact

L. Boutet de Monvel , I. Chueshov, (Doklady Ac.Nauk.1997, CMPDE 1998, NA 1998);
I. Chueshov, A. Rezounenko (CRAS 1995, CMPDE 1997, CPAA 2015, NA 2015.) I. Ryzhykova (Comm. Math. Physics, 2004, JMAA 2006)

Why rotational forces model is inappropriate for studying stabilization of flutter.?

FACT: Experimental finding: **dispersion [flow] "stabilizes" the structure.**

What does it say about modeling.?

This is impossible with rotational forces accounted for; i.e.Δu_{tt} added.

$$u_{tt} + \gamma \Delta u_{tt} + \Delta^2 u + f(u) = tr_U(\Phi)$$

HERE IS WHY.

Hidden stabilizing effect of the flow.

$$u_{tt} + \Delta^2 u + f(u) = \mathbf{tr_U \Phi}$$

becomes nonlinear PDE with a delay

$$u_{tt} + \Delta^2 u + f(u) = -\mathbf{tr_U(u)} + \mathbf{q(u,t,x,y)}$$

$$q(u,t,x,y) = \int_0^{t^*} ds \int_0^{2\pi} d\theta D^2(u(x-(U+sin\theta)s, y-scos\theta, t-s), DELAY$$

$$u_{tt} + \underbrace{u_t}_{stabilizes} + \Delta^2 u + f(u) = -\underbrace{Uu_x + q(u,t,x,y)}_{destabilizes}$$

Rotational model:

$$u_{tt} - \gamma \Delta u_{tt} + \Delta^2 u + f(u) = -\mathbf{tr_U \Phi} \text{ becomes}$$

$$u_{tt} - \gamma \Delta u_{tt} + \Delta^2 u + f(u) = -\mathbf{tr_U(u) + q(u, t.x.y)}$$

$$u_{tt} - \gamma \Delta u_{tt} + \underbrace{u_t}_{does\ \ not\ \ stabilize} + \Delta^2 u + f(u) = \underbrace{U u_x + q(u, t, x, y)}_{destabilizes}$$

$$u_{tt} - \gamma \Delta u_{tt} + \underbrace{u_t - \gamma \Delta u_t}_{stabilizes} + \Delta^2 u + f(u) = \underbrace{U u_x + q(u, t, x, y)}_{destabilizes}$$

Conclusion: To stabilize structure : needs to add Δu_t . Flow does not harvest such term. Flow does not stabilize the rotational model.

Conclusions

1. flow does not stabilize the **rotational model** -confirming the experiment.
2. **Rotational model** yields smoother solutions by adding ONE DERIVATIVE making $f(u)$ compact. Simplifies the analysis **however not physical**.
3. The original model brings aboard **interesting mathematics:**
   - harmonic analysis, $f(u)$ becomes supercritical
   - non dissipative, lack of Lyapunov dynamical system theory: Uu_x
   - PDE dynamics with the delay: $q(u)$ and its destabilizing effects.
   - Lack of compactness, regularity.

Gives rise to NEW Techniques in

- PDE, harmonic and microlocal analysis, weak Hardy spaces,
- Dynamical Systems and Theory of Attractors for Non-Dissipative systems.

Main Results- Overview

- Existence and Hadamard wellposedness of **finite energy solutions** .
- Solutions are bounded for ALL TIMES. - CRITICAL role of **nonlinearity.** (**False for the linearization**)
- **Subsonic case:** with the **feedback control damping** implemented on the structure all solutions stabilize to the **stationary states.**

Conclusion: Flutter can be eliminated

- **Supersonic case:** With the flow data compactly supported all weak solutions of the structure **without any dissipation** converge to a **finite dimensional set** .

Conclusion: Supersonic dynamics becomes smooth and finite dimensional asymptotically.

Previous analysis for regularized- parabolic like models.

Analysis of the original model: Critical role of

- Hyperbolicity-propagation of stability harvested from the flow.
- Nonlinearity at the critical/supercritical level. Compensated compactness/harmonic analysis.
- Sharp Trace Hyperbolic Theory -carriers of propagation
- New tools in dynamical systems and the theory of attractors.

Attractors out of a Thin Air.

Model: Energies

- **Plate** $\mathcal{E}_{pl}(t) = \frac{1}{2}(||u_t||^2 + ||\Delta u||^2 + ||\Delta \mathcal{F}(u)||^2)$

- **Flow**: $\mathcal{E}_{fl}(t) = \frac{1}{2}(||\phi_t||^2 + ||\nabla \phi||^2 - U^2 ||\partial_x \phi||^2)$

- **Interactive**: $\mathcal{E}_{int}(t) = 2U < u_t, \gamma[\phi_x] >_{\partial D}$

$$\mathcal{E}_{pl}(t) + \mathcal{E}_{fl}(t) + \mathcal{E}_{int}(t) = Constant$$

Total Energy -why it is interesting?

- $\mathcal{E}(t) = \mathcal{E}_{pl}(t) + \mathcal{E}_{fl}(t) + \mathcal{E}_{int}(t)$.

Balance of Energy: $\mathcal{E}(t) = \mathcal{E}(s)$ when $k = 0$. **NO DISSIPATION**.

Hidden dissipation - dispersive effects to account for.

- $U < 1$, $\mathcal{E}_{fl} = \frac{1}{2}\left(||\phi_t||^2 + ||\nabla\phi||^2 - U^2||\partial_x\phi||^2\right) \sim ||\phi_t||^2 + ||\nabla\phi||^2$
- $U = 1$, $\mathcal{E}_{fl} \sim ||\phi_t||^2 + ||\partial_z\phi||^2 + ||\partial_y\phi||^2 + 0 \cdot ||\partial_x\phi||^2$
- $U > 1$, $\mathcal{E}_{fl} \sim ||\phi_t||^2 + ||\partial_z\phi||^2 + ||\partial_y\phi||^2 - (U-1)||\partial_x\phi||^2$

Nonlinear Semigroup -Generation

Theorem (J. Webster,NA 2011, Chueshov, I.L, Webster 2013 JDE)

- *Flow-structure interaction generates a continuous nonlinear semigroup*

$$S_t : H \to H = H_0^2(\Omega) \times L_2(\Omega) \times H^1(D) \times L_2(D)$$

- *Semigroup S_t is* **bounded** *for all $t > 0$ ($U < 1$).*
- *For compatible and suitably smooth initial data the corresponding solutions are* **smooth and global**.

Main Result - Subsonic and Supersonic

Theorem (Finite dimensional attracting set, CMPDE 2014)

- *Let $U \neq 1$, $k \geq 0$. Consider plate solutions in $H_{pl} = H_0^2(\Omega) \times L_2(\Omega)$. Then, there exists a compact set $U \in H^3 \times H^2 \subset H_{pl}$ of finite fractal dimension such that*

$$lim_{t\to\infty} dist\{(u(t), u_t(t)), U\} =$$

$$lim_{t\to\infty} inf_{u_0,u_1\in U}[||u(t) - u_0||^2_{2,\Omega} + ||u_t(t) - u_1||^2_{0,\Omega}] = 0$$

for all **compactly supported initial conditions** *corresponding to the flow.*

Strong Stability- Subsonic case

$\mathcal{N}$ denotes stationary solutions. Assume it is **finite.** (generically true. Can be eliminated :Haraux, Lojasiewicz lemma:)

Theorem (Stability, $U < 1$ and $k > k_0 > 0$, SIMA 2016)

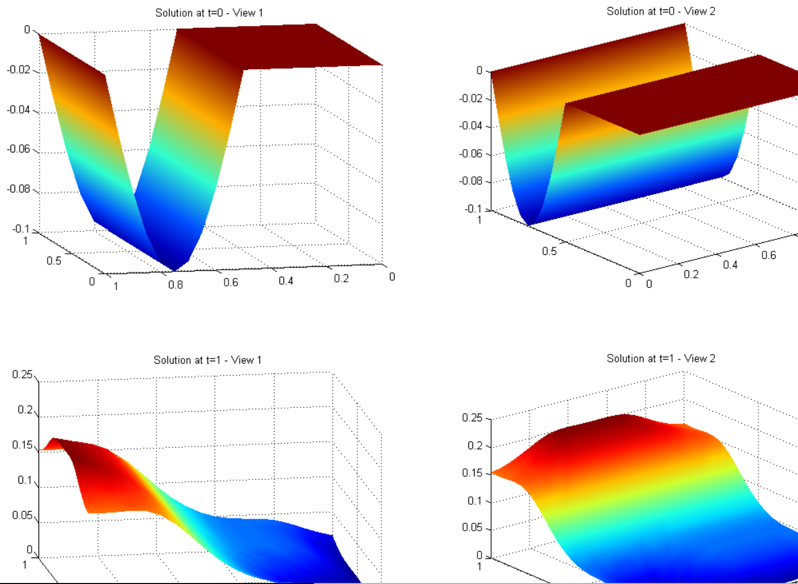
Let $U < 1$. Then any weak solution with **compactly supported flow initial data** *stabilizes to a stationary set. There exist* $(u_0, u_1, \Phi_0, \Phi_1) \in \mathcal{N}$ *such that for all $R > 0$.*

$$lim_{t\to\infty}||u(t) - u_0||^2_{2,\Omega} + ||u_t(t) - u_1||^2_{0,\Omega} = 0$$

$$lim_{t\to\infty}||\Phi(t) - \Phi_0||^2_{1,B(R)} + ||\Phi_t(t) - \Phi_1||^2_{0,B(R)} = 0$$

where $B(R)$ denotes a ball of radius R.

CONSEQUENCE: **Flutter can be eliminated by applying damping to the structure only. Nonlinear effects are critical**


Solution at t=0 - View 1
Solution at t=0 - View 2
Solution at t=1 - View 1
Solution at t=1 - View 2

PROOFS

THM 1: Wellposedness of Energy Solutions $U \in [0,1) \cup (1,\infty)$
JDE 2013-I. Chueshov,IL, J.Webster

THM 2: Attracting Sets for the Structure $U \in [0,1) \cup (1,\infty)$
CMPDE 2014-I. Chueshov,IL,J. Webster

THM 3: Strong Stability for the Flow-Structure $U \in [0,1)$
SIMA 2016- I.L. J.Webster

Review Paper: Mathematical Theory of Flow Structure Interaction
AMO 2016 -I.Chueshov, E. Dowell, I.L. J. Webster.

Challenges-Why it is interesting.

- Supercritical nonlinearity.
- Why a "natural" decoupling of PDE components does not work?
- Why does a "classical dynamical system methodology " fail?.
- What next ?

Natural PDE decoupling: Good Idea which Does not Work

"Loosing" derivatives: R. Triggiani, D. Tataru : 1991-1993

$\vec{\phi}(0) \longrightarrow$ Flow $\dashrightarrow \left|\vec{\phi}(T)\right|_H \to 0$

Flow $\dashrightarrow (\phi, \phi_t) \in H^{2/3} \times H^{-1/3}$

$$\frac{\partial \phi}{\partial \nu} = u_t + U u_x \in L_2$$

Plate $\dashrightarrow (u, u_t) \in H^{5/3} \times H^{-1/3}$

$$u_{tt} + \Delta^2 u + f(u) = \Phi_t + U\Phi_x \in H^{-1/3}$$

Loss of $\frac{1}{3}$ derivative

Lopatiniski condition violated

Hyperbolic Neumann map

Why are we "loosing" derivatives?

$$\Phi_{tt} = \Delta\Phi \quad \text{in } D \times (0,T), \quad \Phi(0) = \Phi_t(0) = 0$$
$$\partial_\nu \Phi = g \quad \text{on } \partial D \times (0,T)$$

$$g \to (\Phi, \Phi_t) \quad \text{Hyperbolic Neumann map } N_h$$
$$N_h g \equiv (\Phi, \Phi_t), \quad \textit{Lopatinski} \quad \textit{fails}$$

$$N_h : L_2(L_2(\partial D)) \to C(H^{2/3}(D) \times H^{-1/3}(D))$$

Loss of $1/3$ derivative when dimension of D > 1 .

Consequences

When $u_t \in H^1(\Omega)$ [*regularized models*] *then* $g = u_t + Uu_x \in H^1$ and

$$Ng \in K - compact \subset H^1 \times L_2$$

When $u_t \in L_2(\Omega)$ *then* $g \in L_2$ and

$$Ng \in H^{2/3}(D) \times H^{-1/3}(D)$$

Loss of $1/3$ derivative when dimension of D > 1

Wellposedness : Energy function

$$E(t) = E_u(t) + E_\Phi(t) + E_{int}(t)$$

$$E_u(t) = \int_\Gamma [|u_t|^2 + |\Delta u|^2 + |\Delta F(u)|^2] d\Gamma$$

$$E_\Phi(t) = \int_\Omega [|\Phi_t|^2 + \int_\Omega [|\nabla \Phi|^2 - \mathbf{U}|\mathbf{\Phi_x}|^2] d\Omega$$

$$E_{int}(t) = \mathbf{U} \int_\Gamma \Phi u_x d\Gamma$$

Energy balance : $E(t) = E(s)$ for all s, t

- E_Φ may **degenerate** when $U > 1$ (supersonic)

$$\Delta - \mathbf{U} D_x^2 = (1 - \mathbf{U}) D_x^2 + D_y^2 + D_z^2$$

- $E(t)$ is not necessarly positive. (E_{int} **-indefinite**).

Properties of the Energy

- $E_\Phi(t)$ with $U > 1$ is non-positive -"hyperbolic type"
- $E_{int}(t)$ has indefinite sign , however we have energy balance

$$E_\Phi(t) + E_{pl}(t) + E_{int}(t) = Constant, t \in R$$

Bad Energy, Good Energy Balance

Supersonic energy

$$E(t) = E_u(t) + E_\Phi(t) + E_{int}(t)$$

$$E_u(t) = \int_\Omega [|u_t|^2 + |\Delta u|^2 + |\Delta F(u)|^2], \quad E_{int}(t) = U \int_\Omega \Phi_x(t) u(t)$$

$$E_\Phi(t) = \int_\Omega [|\Phi_t(t) + U \frac{\partial}{\partial x} \Phi(t)|^2 + |\nabla \Phi(t)|^2] d\Omega$$

Energy relation : $E(t) = E(s) - \mathbf{U} \int_s^t \int_\Gamma (u_t \Phi_x|_\Gamma + U u_x \Phi_x|_\Gamma) dxds$

Good Energy, Bad Energy Balance

Properties of supersonic energy

- Dissipation integral of indefinite sign and not defined for finite energy solutions -

$$< u_t, \Phi_x >_{\partial D}$$

$\Phi_x|_{\partial D}$ **not defined in** $L_2(\partial D)$ for $\Phi \in H^1(D)$

$$< u_t, \Phi_x >_{\partial D} \quad \sim L_2 \cdot H^{-1/2}????$$

Loss of dissipativity and loss of regularity. NOT A GOOD SPELL.

Plate-Structure. Nonlinearity becomes supercritical ($u_t \in L_2$)

Flow- Interface traces not defined on the energy space .

Harmonic Analysis and Microlocal Analysis Enter the Game.

1. **to deal with super linearity of the plate motion** and
2. **to propagate stability harvested by the sheer flow**

Prove Two Regularity Results : (1) for the flow and (2) for the structure

For the flow

Difficulty reduces to

$$\int_0^t \int_\Gamma u_t \Phi_x|_\Gamma.$$

for $u_t \in L_2(\Gamma)$, $\Phi_x \in L_2(\Omega)$ the integrand $L_2 \cdot H^{-1/2}$ is NOT DEFINED.
However It turns out that $\int_0^t \int_\Gamma u_x \Phi_t|_\Gamma < \infty$ **because**

Lemma (Trace Regularity $\rightarrow$ compensated compactness)

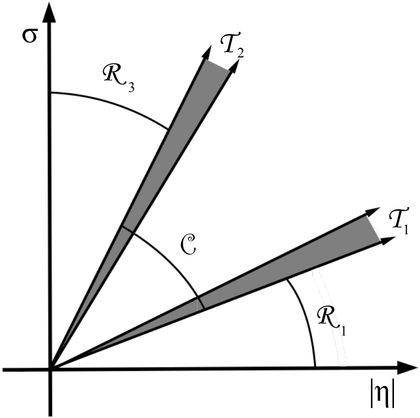
(Microlocal)

$$\Phi \in H^1(Q), \text{ and } \partial_\nu \Phi \in L_2(\Sigma) \Rightarrow \Phi_t|_\Gamma \in L_2(0,T; H^{-1/2}(\Gamma))$$

$\Phi_t \in L_2(\Omega)$ does not allow for application of the trace operator.

$\Phi_t|_\Gamma \in L_2(H^{-1/3}(\Gamma))$. For a classical wave eq with Neumann L_2 data
I.L. R. Triggiani, JDE 1989, D. Tataru 1995, Ann, Sc. Norm

Remarks on Proof: Hidden Regularity/Microlocal III



In $\mathcal{R}_3$-hyperbolic - L_2 regularity, In $\mathcal{R}_1$-elliptic - L_2 regularity
In $\mathcal{C}$-characteristic - $H^{-1/2}$ regularity -loss of regularity.

For the structure:
Harmonic analysis- supercritical nonlinearity

$$u_{tt} + \Delta u = [\mathcal{F}(u), u], \quad \mathcal{F}(u) = \Delta^{-2}[u, u]$$

$$[u, v] : H^2 \times H^2 \to L_1 \subset H^{-\epsilon} \Rightarrow$$
$$\mathcal{F}(u) : H^2 \times H^2 \to H^{3-\epsilon} \Rightarrow$$
$$[\mathcal{F}(u), u] : H^2 \times H^2 \to H^{-\epsilon}, \epsilon > 0$$

Theorem (I.L. D.Tataru, 1994)

$$[u, v] : H^2 \times H^2 \to H_1[Hardy = F^{1,0}] \Rightarrow$$
$$\mathcal{F}(u) : H^2 \times H^2 \to W^{2,\infty} \Rightarrow$$
$$[\mathcal{F}(u), u] : H^2 \times H^2 \to L_2$$

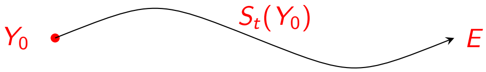
There is **no loss** of ϵ. harmonic analysis+compensated compactness

Strong Stability

S_t is **strongly stable** to an equilibrium in $N = \{E \in H, A(E) = 0\}$

- $\forall Y_0 \in H, S_t(Y_0) \to E$ when $t \to \infty$, for $E \in N$

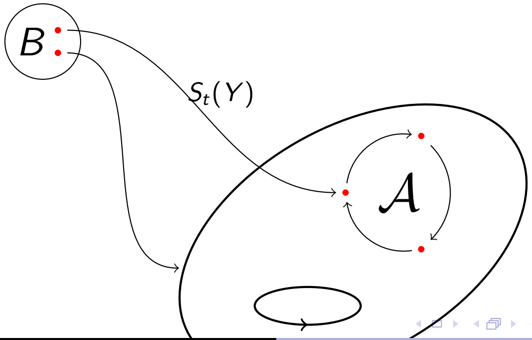
Strong Stability



$S_t(Y_0) \to E$ when $t \to \infty$

Attractors

- $S_t(\mathcal{A}) = \mathcal{A}, \ \forall t > 0$-invariance under the flow
- For any $B \in H$, $d(S_t(B), \mathcal{A}) \to 0, t \to \infty$-property of attraction.



First Look at Strong Stability:
ONE NEEDS

1. **Energy Identity for:** $Y_t = AY + F(Y)$

2. **Dissipativity of** $A(Y)$
3. **Strict Lyapunov function (Unique Continuation Property):** $\frac{d}{dt}S_t(Y) = 0 \to Y = 0$
4. **Compactness**

Nr 2-4 Fail for the flow-structure

Existence of global attractors

Principle of Decomposition

$S_t = U_t + K(t)$
U_t **uniformly** stable to zero equilibrium
$K(t)$ **compact** operator on H for some $t > 0$

Then there exists a global (compact) attractor for S_t.

Absorbing Ball (dissipation) + asymptotic smoothness (compactness) $\Rightarrow$ existence of global attractor.

Babin, Hale, Ladyzhenskaya, Mane, Sell, Temam, Vishik....

ONE NEEDS

1. **Uniform stability to zero for e^{At}**
2. **Compactness of F or Smoothing property of e^{At}**

1-2 fail for flow-structure

Due to hyperbolic nature of the problem and lack of dissipativity.
Lack of smoothing parabolic effects

What to do next????

Long time behavior: STRATEGY

1. **STEP 1:** $k > 0$. Strong convergence to the set $\mathcal{N}$ of orbits driven by **smooth structural** data
   - Use **dispersion** estimates for the flow driven by the initial conditions
   - Analysis of the coupling via Neumann map: "tour de force " loosing derivatives . **Strong stability for smooth** initial data. PDE decoupling ($k > 0$) .

$$S_t(Y_r) \to \mathcal{N}, Y_r \in D(A)$$

2. **STEP 2:** $k > 0$ Uniform Hadamard sensitivity uniformly in $t > 0$ when $||Y_r - Y|| \leq \epsilon$.

$$||S_t(Y_r) - S_t(Y)|| \leq \epsilon c(\int_0^t ||u_t||)$$

Controlling the **rate of attraction** $||u_t|| \in L_1$**?**. We only know $||u_t|| \in L_2$.

- **STEP 3 :** $k \geq 0$ Back to the structure. Construct **attractor** $\mathcal{A}$ for the structure only.
  Big Gun. **No dissipation, no compactness**. But "hidden" dissipation harvested from the flow and "hidden" compactness.
  - $U(t)(B) \to \mathcal{A}$. Prove **smoothness** on that attractor $\mathcal{A}$.
  - **Tool**: **Quasistability estimate**. Use **backward invariance**.

$$|S_T^U(u) - S_T^U(v)|_H \leq 1/10|u - v|_H + C_T sup_{0,T}|S_t^U u - S_t^U v|_{H_1}$$

for $u, v \in \mathcal{A}$, $H \subset H_1$ compact embedding. $H_1 \sim [D(A^{\epsilon})]'$

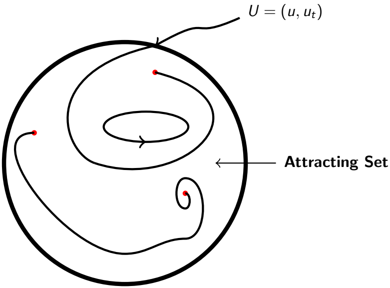
- **STEP 4:**
  - either $U(t)$ enters $\mathcal{A}$ -OK since smooth -go to Step 1.
  - or approaches $\mathcal{A}$. Question: **at which rate?**

- **STEP 5,** $k \geq 0$: Prove an existence of exponential attractor $\mathcal{A}_e \supset \mathcal{A}$.

  $$dist(U(t)B, \mathcal{A}_e) \leq ce^{-\omega t}$$

  The rate is OK, but **smoothness???**. Difficulty: $\mathcal{A}_e$ is only positively invariant.
- **STEP 6,** $k \geq 0$; Prove smoothness of the exponential attractor $\mathcal{A}_e$. Using **quasi stability estimate** for a suitable decomposition of the flow which filters out initial data (Zelik, Vishik). **Attraction at the L_1 rate to a smooth set.** Go back to Step 1.

BIG GUN - for Step 3.
Uniform Convergence with respect to $Y_0 \in B_H$ for the structure only.
Hidden compactness of the delay term $q(u)$ and **hidden** dissipation due to the flow : the term u_t appears "out of the blue".
No dissipation on the flow. Study of systems with delay via Quasistability estimate. The attracting set is smooth, possibly chaotic.



$k = 0$. Hidden stabilizing effect of the flow

$$u_{tt} + \Delta^2 u = [\mathcal{F}(u), u] + p(u, t, x, y)$$
$$p(u, t, x, y) \equiv -(\mathbf{u_t} + \mathbf{U u_x}) + q(u, t, x, y)$$

$$u_{tt} + u_t + \Delta^2 u = [\mathcal{F}(u), u] + U u_x + q(u, t, x, y)$$

$$q(u, t, x, y) = \int_0^{t^*} ds \int_0^{2\pi} d\theta D^2 (u(x - (U + sin\theta)s, y - scos\theta, t - s)$$

$$D^1 = e^{-i\theta} \cdot \nabla^{\perp}_{x,y} = sin\theta \frac{\partial}{\partial x} + cos\theta \frac{\partial}{\partial y}$$

$$t^* = inf\{t, \vec{x}(U, \theta, s) \notin \Gamma, \vec{x} \in \Gamma\}, \vec{x} \equiv (x - s(U + sin\theta), y - scos\theta)$$

Attractor for the plate, $U \neq 1$

Let $k \geq 0$**, no damping** **Big Gun** leads to

Theorem (Chueshov, I.L. Webster CMPDE 2014)

- *Let* $U \neq 1$*. Consider plate solutions in* $H_{pl} = H_0^2(\Omega) \times L_2(\Omega)$*. Then, there exists a compact set* $\mathcal{U} \in H_{pl}$ *of finite fractal dimension such that*

$$lim_{t\to\infty} dist\{(u(t), u_t(t)), \mathcal{U}\} =$$

$$li_{t\to\infty} inf_{u_0,u_1 \in \mathcal{U}}[||u(t) - u_0||^2_{2,\Omega} + ||u_t(t) - u_1||^2_{0,\Omega}] = 0$$

  for all **compactly supported initial conditions** *corresponding to the flow.*
- *There exists* **compact** *"attractor" for the plate .*

The analysis reduced to a finite dimensional invariant set
Determination of the fluttter speed

Uniform Hadamard for the full system

$$Y = (\Phi, \Phi_t, u, u_t)$$

$$|Y(t) - Y_m(t)^2|_H \leq C^{|d|_{L_1}} |Y(0) - Y_m(0)|_H^2$$
$$d(t) = |u_t(t)|_{Y_{pl}} + |u_{m,t}(t)|_{Y_{pl}}$$

We know only that $|u_t|_{L_2(0,\infty)} + |u_{m,t}|_{L_2(0,t\infty} < \infty$

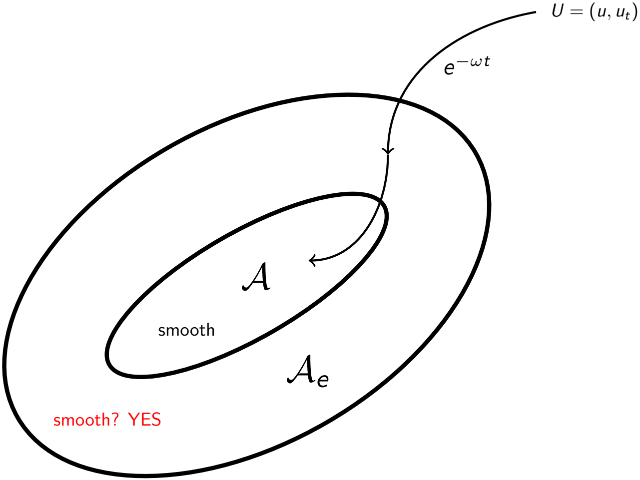
We need $d \in L_1(0,\infty)$ rather than $d \in L_2(0,\infty)$.

Dychotomy Principle

The trajectory $U = (u, u_t)$ EITHER enters the attractor
OR approaches the attractor with $L_1(0, \infty)$ rate.

In the first scenario - previously developed "smooth analysis" applies.
In the second scenario: approximate the trajectory by smooth and L_1 convergent solutions. Leads to **exponential attractors**.

- Exponential attractor $\mathcal{A} \subset \mathcal{A}_e$: convergence to the attractor is exponential. **No information on the smoothness**.
- Regular attractor $\mathcal{A}$: Smooth but **no information on the rate** of convergence.
- A. Miranville and S. Zelik: Survey article in the Handbook on DE -2010. Closing this gap (even for discrete dynamical systems) is in general **open problem**.
- **Finally -Exponential attractor $\mathcal{A}_e$ for the structure is SMOOTH.**



$$\mathrm{dist}(U(t), \mathcal{A}_e) \le Ce^{-\omega t}$$

Back to the flow

- Flow provides **"hidden" dissipation**
- Structure - "plate" provides **"hidden" asymptotic regularity on the attracting set**.
- **Propagating** these properties through the entire system -main challenge of the problem.

SUBSONIC CASE:

Flutter can be eliminated by applying damping to the structure.

SUPERSONIC CASE:

Flow **has stabilizing effect**.With **no damping** on the structure solutions are driven to a finite dimensional set. PDE dynamics reduced to ODE dynamics. Structure of the set : **chaotic, periodic orbits, limit cycles**

Conclusion

Stabilizing effect of the flow exhibited only for a correct model -unregularized and without rotational inertia .

Work in Progress and Open Problems

- **TRANSONIC** CASE. [$U = 1$]. Numerical evidence of shocks . Analysis **must account for nonlinearity of the flow**.
- **Kutta Jukovsky** boundary conditions.

$$\phi_t + U\phi_x = 0 \quad off \quad the \quad wing.$$

  Mathematical interest: **invertibility of finite Hilbert transforms.**
  L_p **theory for** $p \neq 2$**.**
  Chueshov, I.L, Webster - DCDS 2014.
- **Free -clamped** boundary conditions on the plate. Experiments indicate hysteresis -not predicted by the present model. Use **NSE** to model the flow.
- **Nonlinear Flow equation**: NS or Euler

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THANK YOU - IGOR