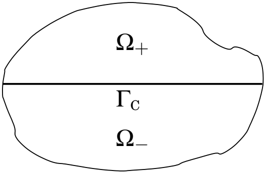
Weierstrass Institute for
Applied Analysis and Stochastics

A model for rate-independent, brittle delamination in thermo-visco-elasticity

joint work with Riccarda Rossi

Marita Thomas

Modeling of delamination along an interface



irreversible crack evolution along a prescribed surface

Γ_C: (flat) interface with evolving delamination (= crack initiation & growth)

$\Omega := \Omega_- \cup \Gamma_C \cup \Omega_+$

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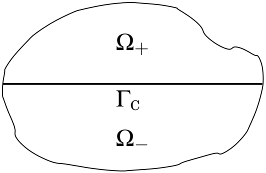

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Modeling approach: Continuum damage mechanics: [Frémond82,87]

delamination variable $z : [0,T] \times \Gamma_C \to [0,1]$ volume fraction of active bonds

$\mathrm{crack}(t) := \{x \in \Gamma_C, z(t,x) = 0\}$ $\forall t_1 < t_2 : z(t_2) \leq z(t_1)$ a.e. on Γ_C

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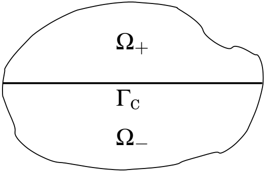

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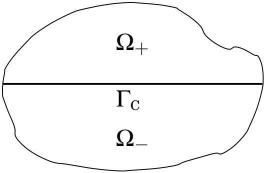
displacement $u : [0,T] \times \Omega$, $[\![u]\!]$: jump of u across Γ_C

brittle delamination: $\forall t \in [0,T]$: $z(t)[\![u(t)]\!] = 0$ a.e. on Γ_C

adhesive contact: $\forall t \in [0,T]$: $\underbrace{z(t)[\![u(t)]\!] \neq 0 \text{ allowed on } \Gamma_C}_{\text{penalized by energy term}}$

[Kočvara/Mielke/Roubíček06,Roubíček/Scardia/Zanini09, Bonetti/Bonfanti/Rossi08,09]

Modeling of delamination along an interface



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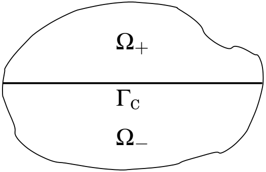
nonpenetration condition: $[\![u]\!] \cdot \mathrm{n} \geq 0$ a.e. on Γ_C

Modeling of delamination along an interface



irreversible crack evolution along a prescribed surface

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Plan of the talk:

1. Mathematical modeling of delamination
   - Fully rate-independent evolution of brittle delamination: energetic formulation
   - Extention of the model by rate-dependent effects
2. Adapted energetic formulation & suitable regularizations
3. Main result & mathematical tools

Rate-independent evolution of brittle delamination

State variable $q=(u,z)$:

$u:[0,T]\times\Omega\to\mathbb{R}^d$ displacement, $e(u)=\frac{1}{2}(\nabla u+\nabla u^\top)$ lin. strain

$z:[0,T]\times\Gamma_\mathrm{C}\to[0,1]$ delamination variable

Energy functional

$$\Phi(t,q):=\int_{\Omega\backslash\Gamma_\mathrm{C}}W(e(u))\mathrm{d}x+\int_{\Gamma_\mathrm{C}}\Big(I_{\{z[\![u]\!]=0\}}([\![u]\!],z)+I_{[0,1]}(z)+I_{\{[\![u]\!]\cdot\mathrm{n}\geq0\}}([\![u]\!])-a_0z\Big)\mathrm{d}s-\langle F(t),u\rangle$$

indicator function of set/constraint C: $I_C(y):=\begin{cases}0 & \text{if } y\in C\\ \infty & \text{otherwise}\end{cases}$

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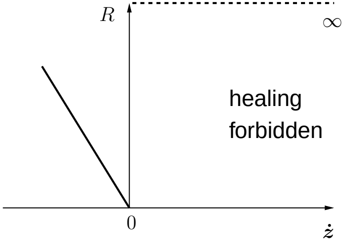
Dissipation potential:

$$\mathscr{R}_1(\dot z)=\int_{\Gamma_\mathrm{C}}R_1(\dot z(x))\mathrm{d}s \qquad R_1(\dot z):=\begin{cases}a_1|\dot z| & \text{if } \dot z\leq0\\ \infty & \text{else}\end{cases}$$

with $a_1>0$



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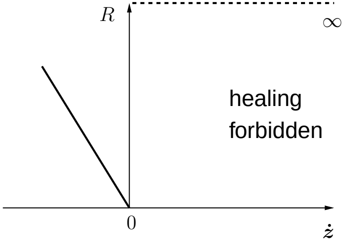
Rate-independence ⇔ 1-homogeneity:
$\mathscr{R}_1(0)=0$ and $\forall\lambda>0\,\forall v:\mathscr{R}_1(\lambda v)=\lambda\mathscr{R}_1(v)$



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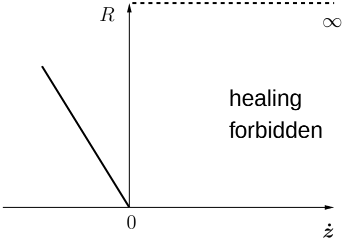

Rate-independence ⇔ 1-homogeneity:

$\mathscr{R}_1(0)=0$ and $\forall\lambda>0\,\forall v:\mathscr{R}_1(\lambda v)=\lambda\mathscr{R}_1(v)$

$$\Downarrow$$

Dissipation distance: $\mathscr{D}_1(z_1,z_2)=\mathscr{R}_1(z_2-z_1)$

Rate-independent evolution of brittle delamination II

Subdifferential formulation:

Find $q \in \mathscr{Q}$ such that $q(0) = q_0$ and $\quad 0 \in \partial_q \Phi(t,q) + \partial_{\dot{q}} \mathscr{R}_1(\dot{q})$

Rate-independent evolution of brittle delamination II

Subdifferential formulation:

Find $q \in \mathscr{Q}$ such that $q(0) = q_0$ and $\quad 0 \in \partial_q \Phi(t,q) + \partial_{\dot q} \mathscr{R}_1(\dot q)$

refers to the system (formally):

momentum balance for u in $(0,T) \times \Omega \backslash \Gamma_{\mathrm{C}}$:

$$-\,\mathrm{div} \mathrm{D}_e W(e(u)) = F \qquad + \text{BCs on } (0,T) \times \partial\Omega + \text{IC}$$

& flow rule for z on $(0,T) \times \Gamma_{\mathrm{C}}$:

$$0 \in \partial_{\dot z} R_1(\dot z) + \partial_z I_{\{z[\![u]\!]=0\}}([\![u]\!], z) + \partial_z I_{[0,1]}(z)\big) - a_0 \qquad +\text{IC}$$

& constraint for (u,z) on $(0,T) \times \Gamma_{\mathrm{C}}$:

$$[\![\mathrm{D}_e W(e(u))]\!]\mathrm{n} = 0$$

$$0 \in \partial_u I_{[z[\![u]\!]=0]}([\![u]\!], z) + \partial_u I_{[[\![u]\!]\cdot \mathrm{n} \geq 0]}([\![u]\!]) + \mathrm{D}_e W(e(u))\mathrm{n}$$

due to brittle constraint and nonpenetration

Rate-independent evolution of brittle delamination II

Subdifferential formulation:

Find $q:[0,T]\to\mathscr{Q}$ such that $q(0)=q_0$ and $\quad 0\in\partial_q\Phi(t,q)+\partial_{\dot q}\mathscr{R}_1(\dot q)$

Alternative, weaker problem formulation due to 1-homogeneity of $\mathscr{R}_1$

Find an **energetic solution** for $(\mathscr{Q},\Phi,\mathscr{R}_1)$

Definition [Mielke&Co]**:** $q:[0,T]\to\mathscr{Q}$ is an **energetic solution** to $(\mathscr{Q},\Phi,\mathscr{R}_1)$, if for all $t\in[0,T]$ it holds $\partial_t\Phi(\cdot,q(\cdot))\in L^1((0,T))$, $\Phi(t,q(t))<\infty$ and:

$$\begin{cases}(\mathrm{S})\ \text{Stability}: & \text{for all } \tilde q\in\mathscr{Q}:\ \Phi(t,q(t))\le\Phi(t,\tilde q)+\mathscr{R}_1(\tilde z-z(t)),\\ (\mathrm{E})\ \text{Energy balance}: & \Phi(t,q(t))+\mathrm{Diss}_{\mathscr{R}_1}(z,[0,t])=\Phi(0,q(0))+\int_0^t\partial_t\Phi(\xi,q(\xi))\mathrm{d}\xi,\end{cases}$$

where $\mathrm{Diss}_{\mathscr{R}_1}(z,[s,t]):=\sup_{\text{all part. of }[s,t]}\sum_{j=1}^N\mathscr{R}_1(z(\xi_j)-z(\xi_{j-1}))$.

Rate-independent evolution of brittle delamination II

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[Roubíček/Scardia/Zanini09]: Existence of energetic solutions for brittle delamination by approximation with adhesive contact: $\frac{k}{2}z|[\![u]\!]|^2\to I_{\{z[\![u]\!]=0\}}([\![u]\!],z)$ as $k\to\infty$!

Extention of the model:

brittle, rate-independent delamination & nonpenetration

& visco-elastic material & thermal effects

Analogous strategy as in the fully rate-independent case:

adhesive model from [Rossi/Roubíček10] $\xrightarrow{?}$ brittle model

Brittle delamination & nonpenetration

state variables: $u : [0,T] \times \Omega \to \mathbb{R}^d$ displacement, $e(u) = \frac{1}{2}(\nabla u + \nabla u^\top)$ lin. strain

$z : [0,T] \times \Gamma_C \to \{0,1\}$ delamination variable

$(u, z \quad)$ coupled in system given by

momentum balance$(e(u) \qquad\qquad)$

& flow rule$(\dot{z})$

& constraint$(z, [\![u]\!])$ on Γ_C

due to brittle constraint & nonpenetration

+ BCs + ICs

rate-independent evolution of z

$\Rightarrow$ energetic formulation via global stability & energy balance

Brittle delamination & nonpenetration & viscosity & therm. effects

state variables: $u:[0,T]\times\Omega\to\mathbb{R}^d$ displacement, $e(u)=\frac{1}{2}(\nabla u+\nabla u^\top)$ lin. strain

$z:[0,T]\times\Gamma_\mathrm{C}\to\{0,1\}$ delamination variable

$\theta:[0,T]\times\Omega\to(0,\infty)$ absolute temperature

(u,z,θ) coupled in system given by

momentum balance$(e(u),e(\dot{u}),\theta)$

& heat equation$(e(u),e(\dot{u}),\dot{\theta},\theta)$

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rate-independent evolution of z but viscous evolution of u: $R_2(e(\dot{u}))=|e(\dot{u})|^2$,

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rate-independent evolution of z but viscous evolution of u: $R_2(e(\dot{u}))=|e(\dot{u})|^2$,
$\Rightarrow$ no energetic formulation via global stability & energy balance

But: adapted energetic formulation of system in terms of 4 conditions [Roubíček10]:

1. weak momentum balance (for u)
2. weak enthalpy (in)equality (for θ)
3. semistability (for z)
4. mechanical energy 'balance'

Delamination in viscoelastic materials with thermal effects

adhesive model from [Rossi/Roubíček10] $\xrightarrow{\checkmark}$ brittle model

2. Adapted energetic formulation & sufficiently regularized models

3. Main result and tools

Weak formulation of the heat equation$(e(u), e(\dot{u}), \dot{\theta}, \theta)$

$$c_{\mathrm{v}}(\theta)\dot{\theta} + \operatorname{div}\mathbb{J}(e(u),\theta) = 2R_2(e(\dot{u})) - \theta\mathbb{C}\mathbb{E}e(\dot{u}) + H \qquad \text{in } [0,T]\times\Omega\backslash\Gamma_{\mathrm{C}}$$

- H external heat source, $\mathbb{C}$, $\mathbb{E}$ sym., pos. def. fourth order tensors
- Fourier's law for heat flux: $\mathbb{J}(e,\theta) = -\mathbb{K}(e,\theta)\nabla\theta$
- heat capacity $c_{\mathrm{v}}(\theta)$
- viscous dissipation: $R_2(\dot{e}) := |\dot{e}|^2$

Weak formulation of the heat equation$(e(u), e(\dot{u}), \dot{\theta}, \theta)$

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Idea: Use time-discretization to prove existence of energetic sol.s for the full PDE-system

Problem: nonlinearity $c_{\mathrm{v}}(\theta)\dot{\theta}$

Weak formulation of the heat equation$(e(u), e(\dot{u}), \dot{\theta}, \theta)$

$$c_{\mathrm{v}}(\theta)\dot{\theta} + \operatorname{div}\mathbb{J}(e(u),\theta) = 2R_2(e(\dot{u})) - \theta\mathbb{C}\mathbb{E}e(\dot{u}) + H \qquad \text{in } [0,T]\times\Omega\backslash\Gamma_{\mathrm{C}}$$

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Problem: nonlinearity $c_{\mathrm{v}}(\theta)\dot{\theta}$

Way out: enthalpy-transformation [Roubíček10]:

enthalpy $w = w(\theta) := \int_0^\theta c_{\mathrm{v}}(\xi)\,\mathrm{d}\xi\,, \qquad \theta = \Theta(w)\,, \qquad \widetilde{\mathbb{K}}(e,w) = \mathbb{K}(e,\Theta(w))/c_{\mathrm{v}}(\Theta(w))\,.$

Reformulate full PDE-system in terms of w

Weak formulation of the heat equation$(e(u), e(\dot{u}), \dot{\theta}, \theta)$

$$\dot{w} - \mathrm{div}(\widetilde{\mathbb{K}}(e(u), w)\nabla w) = 2R_2(e(\dot{u})) - \Theta(w)\mathbb{C}\mathbb{E}e(\dot{u}) + H \quad \text{in } [0,T] \times \Omega\backslash\Gamma_\mathrm{C}$$

- H external heat source, $\mathbb{C}$, $\mathbb{E}$ sym., pos. def. fourth order tensors
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+ BCs on $\partial\Omega$ + conditions on Γ_C involving dissipation $R_1(\dot{z})$

leads to weak ethalpy (in)equality:

$$\langle \zeta(T), w(T) \rangle + \int_Q \widetilde{\mathbb{J}}(e(u), w) \cdot \nabla\zeta - w\dot{\zeta}\,\mathrm{d}x\mathrm{d}t + \int_{\Sigma_\mathrm{C}} \eta([\![u]\!], z)[\![\Theta(w)]\!][\![\zeta]\!]\mathrm{d}s\mathrm{d}t$$

$$\left\{ \begin{matrix} = \\ \geq \end{matrix} \right\} \int_Q (2R_2(e(\dot{u}))\zeta - \Theta(w)\mathbb{C}\mathbb{E} : e(\dot{u})\zeta)\,\mathrm{x}\mathrm{d}t + \int_{\Sigma_\mathrm{C}} \tfrac{1}{2}(\zeta|^+_{\Gamma_\mathrm{C}} - \zeta|^-_{\Gamma_\mathrm{C}})\,\mathrm{d}\xi^S_{\dot{z}}(s, T)$$

$$+ \int_Q H\zeta\,\mathrm{d}x\mathrm{d}t + \int_{\Omega\backslash\Gamma_\mathrm{C}} w_0\zeta(0)\,\mathrm{d}x$$

for all testfcts $\zeta \geq 0$ a.e. adhesive contact '$=$', brittle delamination '$\geq$'

$\mathrm{d}\xi^S_{\dot{z}}(s, T)$ is a measure induced by dissipation $R_1(\dot{z})$

Weak formulation of the momentum balance$(e(u), e(\dot{u}), \theta)$

$$-\operatorname{div}\sigma(u,\dot{u},\theta) = F \quad \text{in } [0,T]\times\Omega\backslash\Gamma_{\mathrm{C}} \qquad (F \text{ applied bulk force})$$

stress $\sigma(u,\dot{u},\theta)$ features viscous response and thermal effects (Kelvin-Voigt rheology)

$$\sigma(u,\dot{u},\theta) := \mathrm{D}_e R_2(e(\dot{u})) + \mathrm{D}_e W_p(e(u)) + \mathbb{C}(e(u) - \mathbb{E}\theta)$$

viscous dissipation: $R_2(\dot{e}) := |\dot{e}|^2$

bulk energy density: $W(e,\theta) := W_p(e) + \frac{1}{2}(e(u) - \mathbb{E}\theta) : \mathbb{C} : (e(u) - \mathbb{E}\theta)$

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viscous dissipation: $R_2(\dot{e}) := |\dot{e}|^2$
bulk energy density: $W(e,\theta) := W_p(e) + \frac{1}{2}(e(u)-\mathbb{E}\theta) : \mathbb{C} : (e(u)-\mathbb{E}\theta)$

+ BCs + constraint$(z, [\![u]\!])$ on Γ_{C}:

$$0 \in \partial_u I_{\{[\![u]\!]\cdot\mathrm{n}\geq 0\}}(u) + \sigma(u,\dot{u},\theta)\mathrm{n} + \partial_u I_{\{z[\![u]\!]=0\}}(z,u)$$

Weak formulation of the momentum balance$(e(u),e(\dot{u}),\theta)$

$$-\operatorname{div}\sigma(u,\dot{u},\theta)=F \quad \text{in } [0,T]\times\Omega\backslash\Gamma_C$$

stress $\sigma(u,\dot{u},\theta)$ features viscous response and thermal effects (Kelvin-Voigt rheology)

$$\sigma(u,\dot{u},\theta):=\mathrm{D}_eR_2(e(\dot{u}))+\mathrm{D}_eW_p(e(u))+\mathbb{C}(e(u)-\mathbb{E}\theta)$$

viscous dissipation: $R_2(\dot{e}):=|\dot{e}|^2$
bulk energy density: $W(e,\theta):=W_p(e)+\frac{1}{2}(e(u)-\mathbb{E}\theta):\mathbb{C}:(e(u)-\mathbb{E}\theta)$

+ BCs + constraint$(z,[\![u]\!])$ on Γ_C:

$$0\in\partial_u I_{\{[\![u]\!]\cdot\mathrm{n}\geq 0\}}(u)+\sigma(u,\dot{u},\theta)\mathrm{n}+\partial_u I_{\{z[\![u]\!]=0\}}(z,u)$$

leads to weak momentum balance as a variational inequality for a.a. $t\in(0,T)$:
$[\![u(t)]\!]\cdot\mathrm{n}\geq 0$, $z(t)[\![u(t)]\!]=0$ on Γ_C and

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$$\geq\int_{\Omega\backslash\Gamma_C}F(t)\cdot(v-u(t))\mathrm{d}x$$

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Weak formulation of the momentum balance $(e(u), e(\dot{u}), \theta)$

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double constraint ⇒ approximate brittle constraint $z(t)[\![v]\!] = 0$ by adhesive contact

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\+ BCs + constraint$(z, [\![u]\!])$ on Γ_C:

$$0 \in \partial_u I_{\{[\![u]\!]\cdot\mathrm{n}\geq 0\}}(u) + \sigma(u,\dot u,\theta)\mathrm{n} \;+ kz[\![u]\!]$$

leads to weak momentum balance as a variational inequality for a.a. $t \in (0,T)$:
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approximate brittle constraint $z(t)[\![v]\!] = 0$ by adhesive contact

Weak formulation of the flow rule$(\dot{z})$

$$0 \in \partial_{\dot{z}} R_1(\dot{z}) + \partial_z \phi^s(z, [\![u]\!]) \qquad \text{on } \Gamma_{\rm C}$$

- rate-independent dissipation $R_1(\dot{z}) = -a_1\dot{z} + I_{(-\infty,0]}(\dot{z})$
- surface energy density $\phi^s(z, [\![u]\!]) = -a_0 z + I_{[0,1]}(z) + I_{[z[\![u]\!]=0]}(z, [\![u]\!])$

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weak formulation: semistability

$$\forall t \in [0,T], \forall \text{ testfcts } \tilde{z}: \qquad \Phi^s(z(t),[\![u(t)]\!]) \le \Phi^s(\tilde{z},[\![u(t)]\!]) + \mathscr{R}_1(\tilde{z} - z(t))$$

'partial' stability condition, since $u(t)$ (energetic sol.) is fixed!

Recall stability in the (fully) rate-independent setting:

$$\forall t \in [0,T], \forall \text{ testfcts } (\tilde{u},\tilde{z}): \quad \mathscr{E}(t,u(t),z(t)) \le \mathscr{E}(t,\tilde{u},\tilde{z}) + \mathscr{R}_1(\tilde{z} - z(t))$$

Weak formulation of the flow rule$(\dot{z})$ & regularizations

$\forall t \in [0,T]$, $\forall$ testfcts $\tilde{z}$: $\quad \Phi^s(z(t),[\![u(t)]\!]) \leq \Phi^s(\tilde{z},[\![u(t)]\!]) + \mathscr{R}_1(\tilde{z}-z(t))$

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Regularizations:

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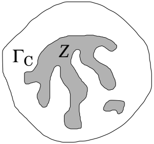
2. enforce $z \in \{0,1\}$ $\quad\rightsquigarrow$ suitable setting:

$\mathrm{SBV}(\Gamma_{\mathrm{C}},\{0,1\}) := \{z : \Gamma_{\mathrm{C}} \to \{0,1\}$, z characteristic fct. of set $Z \subset \Gamma_{\mathrm{C}}$ with $P(Z,\Gamma_{\mathrm{C}}) < \infty\}$

$$\Phi^s(z,[\![u]\!]) + \mathscr{G}_{\mathrm{b}}(z)$$

$\mathscr{G}_{\mathrm{b}}(z) := \mathrm{b}P(Z,\Gamma_{\mathrm{C}})$ perimeter of Z (= total variation of z)



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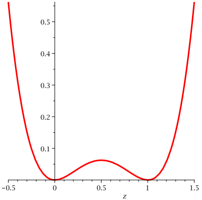
3. Modica-Mortola approximation:

$$\mathscr{G}_m(z) := \begin{cases} \int_{\Gamma_\mathrm{C}} \left(m^2 z^2 (1-z)^2 + \frac{1}{m^2} |\nabla z|^2\right) \mathrm{d}x & \text{if } z \in H^1(\Omega, [0,1]) \\ \infty & \text{otherwise} \end{cases}$$



$\mathscr{G}_m \xrightarrow{\Gamma} \mathscr{G}_\mathrm{b}$ as $m \to \infty$

[Giacomini05] Volume damage → Francfort-Marigo crack model

Weak formulation of the flow rule$(\dot z)$ & regularizations

$\forall t \in [0,T]$, $\forall$ testfcts $\tilde z$: $\quad \Phi^s(z(t),[\![u(t)]\!]) \leq \Phi^s(\tilde z,[\![u(t)]\!]) + \mathscr{R}_1(\tilde z - z(t))$

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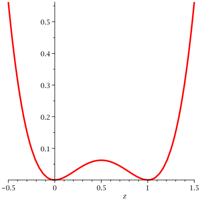

$\mathscr{G}_m \xrightarrow{\Gamma} \mathscr{G}_{\mathrm{b}}$ as $m \to \infty$ adapted to stability cond. for unidirectional $\mathscr{R}_1$: [Th11]

Mechanical energy ’balance’

Energy terms:

- mechanical energy $\Phi(u,z) = \Phi^{\text{bulk}}(u,z) + \Phi^{\text{surf}}(z,[\![u]\!])$
- mechanical bulk energy $\Phi^{\text{bulk}}(u,z) = \int_{\Omega\backslash\Gamma_{\rm C}} W_p(e(u)) + \frac{1}{2}e(u) : \mathrm{C} : e(u)\,\mathrm{d}x$
- mechanical surface energy

$$\Phi^{\text{surf}}(z,[\![u]\!]) := \int_{\Gamma_{\rm C}} \big(-a_0 z + I_{[0,1]}(z) + J_k(z,[\![u]\!]) + I_{\{[\![u]\!]\cdot\mathrm{n}\geq 0\}}([\![u]\!])\big)\,\mathrm{d}s + \mathscr{G}(z)$$

$$J_k(z,[\![u]\!]) = \begin{cases} \frac{k}{2}z|[\![u]\!]|^2 & \text{adhesive, } k\in\mathbb{N} \\ I_{\{z[\![u]\!]=0\}}(z,[\![u]\!]) & \text{brittle, } k=\infty \end{cases} \qquad \mathscr{G}(z) = \begin{cases} \mathscr{G}_m(z) & \text{Modica-Mortola} \\ \mathscr{G}_{\rm b}(z) & \text{SBV (perimeter)} \end{cases}$$

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Mechanical energy ’balance’ for a.a. $t\in(0,T)$:

$$\Phi(u(t),z(t))+\int_0^t\int_{\Omega\backslash\Gamma_{\mathrm{C}}}2R_2(e(\dot u))\mathrm{d}x\mathrm{d}\xi+\mathscr{R}_1(z(t)-z(0))$$

$$\left\{\begin{matrix}=\\ \leq\end{matrix}\right\}\Phi(u(0),z(0))+\int_0^t\int_{\Omega\backslash\Gamma_{\mathrm{C}}}\Theta(w)\mathbb{C}\mathbb{E}:e(\dot u)\mathrm{d}x\mathrm{d}\xi+\int_0^t\int_{\Omega\backslash\Gamma_{\mathrm{C}}}F(t)\cdot\dot u\,\mathrm{d}x\mathrm{d}\xi$$

3. Results: Approximation procedure (viscous, θ-dependent setting)

$$\Phi^{\mathrm{surf}}_{km}(z, [\![u]\!]) := \int_{\Gamma_{\mathrm{C}}} \left(-a_0 z + I_{[0,1]}(z) + I_{\{[\![u]\!]\cdot \mathrm{n} \geq 0\}}([\![u]\!]) + \tfrac{k}{2} z |[\![u]\!]|^2 \right) \mathrm{d}s + \mathscr{G}_m(z)$$

Theorem: Keep $m \in \mathbb{N}$ and $k \in \mathbb{N}$ fixed & assumptions on given data. Then there exist energetic solutions (4 cond.s) to the Modica-Mortola adhesive contact model.

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Limit passage:

Modica-Mortola adhesive contact → SBV-adhesive contact

Theorem: Keep $k \in \mathbb{N}$ fixed & assumptions on given data. As $m \to \infty$, (a subsequence of) the energetic solutions of the Modica-Mortola adhesive contact models approximate an energetic solution of the SBV- adhesive contact model.

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Limit passage $k \to \infty$:

SBV-adhesive contact → SBV-brittle delamination

Main result: Limit passage SBV-adhesive contact → SBV-brittle delamination

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Main result: Limit passage SBV-adhesive contact → SBV-brittle delamination

Theorem: Assumptions on given data. As $k\to\infty$, (a subsequence of) the energetic solutions of the SBV-adhesive contact models approximate an energetic solution of the SBV-brittle delamination model.

Main difficulty: Limit passage in weak momentum balance!

weak momentum balance for SBV-adhesive delamination f.a.a. $t\in(0,T)$:

$[\![u_k(t)]\!]\cdot\mathrm{n}\geq 0$, $\cancel{z(t)[\![u(t)]\!]=0}$ on Γ_C and

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$$+\int_{\Gamma_\mathrm{C}}kz_k(t)[\![u_k(t)]\!]\cdot[\![v-u_k(t)]\!]\,\mathrm{d}s$$

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Idea: construct recovery sequence $(v_k)_{k\in\mathbb{N}}$ for testfct. v such that

$\forall k \in \mathbb{N}: \quad z_k(t)\big[\!\big[v_k\big]\!\big]=0$ a.e. on Γ_C and $v_k \to v$ strongly in $W^{1,p}(\Omega\backslash\Gamma_\mathrm{C},\mathbb{R}^d)$.

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weak momentum balance for SBV-adhesive delamination f.a.a. $t \in (0,T)$:

$[\![u_k(t)]\!]\cdot\mathrm{n} \geq 0$, ~~$z(t)[\![u(t)]\!]=0$~~ on Γ_{C} and

$$\int_{\Omega\backslash\Gamma_{\mathrm{C}}}\big(\mathrm{D}_e W(e(u_k(t)),\Theta(w_k(t)))+\mathrm{D}_e R_2(e(\dot{u}_k(t)))\big):e(v_k-u_k(t))\mathrm{d}x$$

~~$+\int_{\Gamma_{\mathrm{C}}} k z_k(t)[\![u_k(t)]\!]\cdot[\![v_k-u_k(t)]\!]\,\mathrm{d}s$~~

$$\geq +\int_Q F(t)\cdot(v_k-u_k(t))\mathrm{d}x$$

for all sufficiently smooth testfct.s v_k with $[\![v_k]\!]\cdot\mathrm{n}\geq 0$ and $z_k(t)[\![v_k]\!]=0$ on Γ_{C}

Main result: Limit passage SBV-adhesive contact → SBV-brittle delamination

Theorem: Assumptions on given data. As $k \to \infty$, (a subsequence of) the energetic solutions of the SBV-adhesive contact models approximate an energetic solution of the SBV-brittle delamination model.

Main difficulty: Limit passage in weak momentum balance!

Problem: $z(t)[\![v]\!] = 0$, $z_k \overset{*}{\rightharpoonup} z$ in $\mathrm{SBV}(\Gamma_{\mathrm{C}}, \{0,1\})$, but $\int_{\Gamma_{\mathrm{C}}} k z_k(t)[\![u_k(t)]\!] \cdot [\![v]\!] \to ???$

Idea: construct recovery sequence $(v_k)_{k\in\mathbb{N}}$ for testfct. v such that

$\forall k \in \mathbb{N}: \quad z_k(t)[\![v_k]\!] = 0$ a.e. on Γ_{C} and $v_k \to v$ strongly in $W^{1,p}(\Omega\backslash\Gamma_{\mathrm{C}}, \mathbb{R}^d)$.

weak momentum balance for SBV-adhesive delamination f.a.a. $t \in (0,T)$:

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$$\cancel{+\int_{\Gamma_{\mathrm{C}}} k z_k(t)[\![u_k(t)]\!] \cdot [\![v_k - u_k(t)]\!]\,\mathrm{d}s}$$

$$\geq + \int_Q F(t) \cdot (v_k - u_k(t))\mathrm{d}x$$

for all sufficiently smooth testfct.s v_k with $[\![v_k]\!] \cdot \mathrm{n} \geq 0$ and $z_k(t)[\![v_k]\!] = 0$ on Γ_{C}

Reason for $v_k \to v$ strongly in $W^{1,p}$: Mosco-conv. of energy fct. ⇒ G-conv. of derivative

Recovery sequence & support property

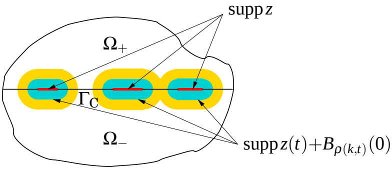
For given testfct. v construct recovery sequence $(v_k)_{k\in\mathbb{N}}$ such that

$$\forall k\in\mathbb{N}: \quad \int_{\Sigma_\mathrm{C}} kz_k [\![u_k]\!]\cdot[\![v_k]\!] = 0$$

and $v_k(t)\to v(t)$ strongly in $W^{1,p}(\Omega\backslash\Gamma_\mathrm{C},\mathbb{R}^d)$

Tool: Support convergence: $\forall t\in[0,T]$

$\operatorname{supp} z_k(t)\subset \operatorname{supp} z(t)+B_{\rho(k,t)(0)}$ for all $k\in\mathbb{N}$ and $\rho(k,t)\to 0$ as $k\to\infty$



$v_k(t)\in W^{1,p}(\Omega\backslash\Gamma_\mathrm{C},\mathbb{R}^d)$ s.th.

$v_k(t)=0$ in $\operatorname{supp} z(t)+B_{\rho(k,t)}(0)$,

$v_k(t)=v(t)$ in $\Omega\backslash\operatorname{supp} z(t)+B_{2\rho(k,t)}(0)$

$[\![v_k(t)]\!]\cdot\mathrm{n}\geq 0$ on Γ_C

[Mielke/Roubíček/Th10]: Let $p>d$. Then, $v_k(t)\to v(t)$ strongly in $W^{1,p}(\Omega\backslash\Gamma_\mathrm{C},\mathbb{R}^d)$.

([Lewis88]: Hardy's inequality)

Support convergence

Support convergence: $\forall t \in [0,T]$

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In general not true: $z_k = 1/k$ on $\Gamma_{\mathrm{C}} \Rightarrow z_k \to z \equiv 0$ unif., but $\operatorname{supp} z = \emptyset$ & $\operatorname{supp} z_k = \Gamma_{\mathrm{C}}$

Excluded for $z_k, z \in \mathrm{SBV}(\Gamma_{\mathrm{C}}, \{0,1\})$

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$$\Updownarrow$$

$$\mathscr{L}^{d-1}((\operatorname{supp} z \backslash \operatorname{supp} z_k) \cup (\operatorname{supp} z_k \backslash \operatorname{supp} z)) \to 0$$

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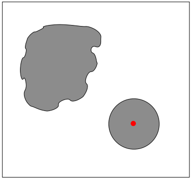

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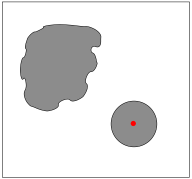

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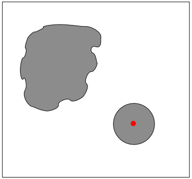

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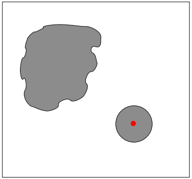

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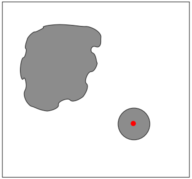

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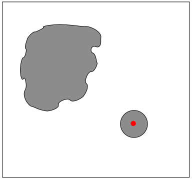

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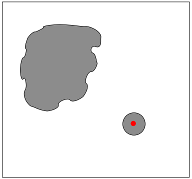

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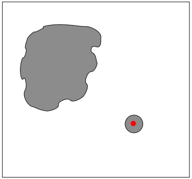

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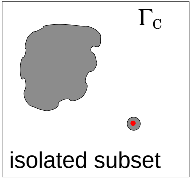

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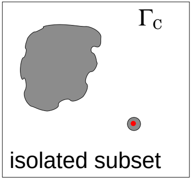

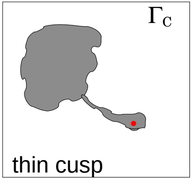

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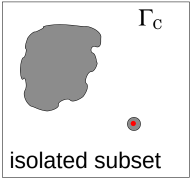
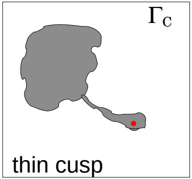


isolated subset

Exclude arbitrarily small sets by semistability!



thin cusp

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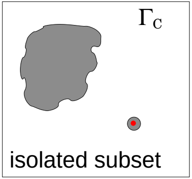
$$\Phi_k(z_k, [\![u_k]\!]) = \int_{\Gamma_{\rm C}} (-a_0 z_k + \tfrac{k}{2} z_k |[\![u_k]\!]|^2)\, \mathrm{d}s + bP(Z_k, [\![u_k]\!])$$



Exclude arbitrarily small sets by semistability!

z_k semistable charact. fct. of Z_k, $A \subset Z_k$ isolated

test semistability with $\tilde{z}$ charact. fct. of $Z_k \backslash A$:

$\Phi_k(z_k, [\![u_k]\!]) \leq \Phi_k(\tilde{z}, [\![u_k]\!]) + \mathscr{R}_1(\tilde{z} - z_k)$

$\Rightarrow bP(A, \Gamma_{\rm C}) \leq (a_0 + a_1) \mathscr{L}^{d-1}(A)$

$$\frac{b}{(a_0+a_1)} \leq \frac{\mathscr{L}^{d-1}(A)}{P(A,\Gamma_{\rm C})} \leq c_{d-1} \mathscr{L}^{d-1}(A)^{1/(d-1)}$$

isoperimetric inequality

R. Rossi and M. Thomas:
From an adhesive to a brittle delamination model in thermo-visco-elasticity,
WIAS-Preprint 1692.

Thank You!