****

PDEs for multiphase advanced materials – ADMAT2012

Palazzone – Cortona (AR) – September 17–21, 2012

****

Scientific and Organizing Committe:

Elena Bonetti (Università degli Studi di Pavia)
Pierluigi Colli (Università degli Studi di Pavia)
Elisabetta Rocca (Università degli Studi di Milano)

`Web-page: http://www-dimat.unipv.it/pier/admat2012.html`

Sponsored by

- *INDAM - Istituto Nazionale di Alta Matematica "F. Severi"*
- *FP7-IDEAS-ERC-StG #2568722010 "EntroPhase"*

BOOK OF ABSTRACTS

MAIN LECTURES

A two-scale problems as mathematical model for sulfate attack in sewer pipes

Toyohiko Aiki

Japan Women's University, Tokyo (Japan)

This is a joint work with Tasnim Fatima and Adrian Muntean (TU Eindhoven, the Netherlands). In this talk we treat the solvability of a semilinear reaction-diffusion system is a two-scale model of concrete corrosion in sewer concrete pipes. It consists of three partial differential equations which are mass-balances of concentrations, as well as, one ordinary differential equation tracking the damage-by-corrosion. The system is semilinear, partially dissipative, and coupled via the solid-water interface at the microstructure (pore) level. Herein we prove the global-in-time existence and uniqueness of a suitable class of positive and bounded solutions. Because of lock of compactness it is necessary to define a solution of the two-scale model by using the variational inequality.

An abstract existence theorem to parabolic systems

Hans Wilhelm Alt

Technical University Munich, Munich (Germany)

In the talk I will refer to the paper H.W.Alt: An abstract existence theorem for parabolic systems. CPAA 11, pp. 2079-2123 (2012) and I will explain the main features of it. It is based on the abstract elliptic theory developed in the 1970's, so it is mainly concerned with the parabolic term in the system. It is essential that the ellipticity assumption is made for a quantity u, where the parabolic quantity v=b(u) is only a monotone function of u. Systems with these properties are well known in physical applications, and I will refer to such systems in my talk.

Consistent n-phase Cahn-Hilliard systems and applications to multiphase flows

Franck Boyer

University Paul Cézanne

In this talk, I will present recent developments of consistent Cahn-Hilliard systems for an arbitrary number of components. Through the coupling with the Navier-Stokes equation, these systems are used to simulate multiphase flows. I will discuss some numerical issues that are to be solved in order to perform such simulations. This is a joint work with S. Minjeaud.

Blowup and stationary states in aggregation equations

José Antonio Carrillo

Barcellona (Spain)

I will discuss nonlocal integral continuity equations for which notrivial stationary states show interesting properties. We will discuss their nonlinear stability/instability and the dimension of their support. They lead to interesting questions of calculus of variations in measure settings. Related problems in 1D will be discussed in relation to some simplified models in fracture dynamics and fluid mechanics.

Cell motility

Antonio DeSimone

Sissa, Trieste (Italy)

The talk will review the fundamentals of biological fluid dynamics, applications to the engineering of micro-robots, the geometric structure underlying the mathematics of low Reynolds number swimming, and numerical algorithms for optimal control. We will discuss the implications on the analysis of the swimming strategies of microscopic swimmers and recipes to optimize the strokes of engineered micro-devices.

Sharp limits of diffuse interface models in the context of energy storage problems

Wolfgang Dreyer

WIAS, Berlin (Germany)

The lecture addresses interface problems in the context of Lithium-ion batteries. We choose two different problems to exhibit the subtle correspondence between sharp and diffuse interface models. During fast charging of the battery a two-phase system develops in the storage electrode. We describe the phenomenon by a viscous Cahn-Hilliard model and carry out a formal limit leading to a sharp interface model. It turns out that the Cahn-Hilliard equation, which implies a non-negative entropy production in the bulk phases, may induce a negative interfacial entropy production in the sharp limit. A further interface problem concerns the interface between the electrode and the adjacent electrolyte. The electrolyte is described by generalized Nernst-Planck equations developing an interesting boundary layer at the electrode. While we meet incompressibility and charge neutrality in the bulk of the electrolyte, we show that both properties vanish in the boundary layer.

Biomembrane modelling and Reaction and diffusion models for cell motility

Charlie Elliott

Warwick, UK

In this talk we describe some mathematical models in cell biology relating to geometric membranes and cell motility which involve surface partial differential equations and present some computational methods and numerical experiments.

Mathematical modelling of migration and integration

Mauro Fabrizio

Università di Bologna

For studying the integration of two different ethnic populations, we compare this evolution with that of a mixture of two fluids. For this model we consider the concentration of only one species, whose evolution will be described by a Cahn-Hilliard equation. While the separation between the two phases will be controlled by the educational levels of two components. Finally, we assume the homogenization phase occurs when the mean of two cultural levels is greater of a critical value.

Motion of a solid with large deformations

Michel Frémond

Roma Tor Vergata (Italy)

We study the motion of a solid with large deformations. Two new elements are introduced:

1. the solid may be loaded on its surface by needles, rods, beams, plates,.. In order to sustain these external actions, it is needed a body third gradient theory;
2. elongation matrix $\mathbf{W}$ of the polar decomposition has to be symmetric. This is an internal constraint which introduces a reaction stress, an antisymmetric matrix $\mathbf{A}$. This reaction matrix intervenes in the angular and linear equations of motion.

We prove that there exist a motion, i.e., a position function $\Phi(a,t)$, depending on initial position a and on time t, which satisfies the complete equations of mechanics. This motion is local in time because it may be interrupted by crushing resulting in a discontinuity of velocity with respect to time, i.e., a collision within the solid. This is a joint work with Elena Bonetti and Pierluigi Colli.

The effect of a thin layer of heterogeneities in an elastic structure

Giuseppe Geymonat

LMS, Ecole Polytechnique (France)

In many industrial situations a 3D structure, linearly elastic for simplicity, may contain heterogeneities periodically distributed along a 2D planar surface. These heterogeneities can be of different types (e.g. holes, elastic inclusions,...) and/or of different geometries (e.g. spherical, cylindrical,...). I will survey some work in progress and the two methods used: (i) a more formal one (matched asymptotic expansions) and (ii) a more rigorous one based on variational convergence

---

Recent results on a singular and possibly degenerate Cahn-Hilliard type system

Gianni Gilardi

University of Pavia, Pavia (Italy)

A singular viscous Cahn-Hilliard type system is presented. A number of results in several directions (well-posedness, long-time behavior, asymptotic analysis with respect to a small parameter, optimal control) obtained in collaboration with P.Colli, J.Sprekels, and P.Podio-Guidugli is already known. The present talk deals with a proper generalization of the system that allows possible degeneracy besides singularity and regards some recent results.

Free energies and phase transitions in materials with hysteresis

Claudio Giorgi

University of Brescia, Brescia (Italy)

In the literature we can find a lot of mathematical models for magnetic hysteresis, many of which are thorough and physically sound (see, for instance, [4]). Some of them, usually referred to as Duhem models, are quite easy to handle and lot well for applications (we quote, for instance, [1] and [2]). Nevertheless, to the best of our knowledge, none of them is so exhaustive to describe the temperature-induced phase transition between the paramagnetic and the ferromagnetic regime. Recently, some efforts have been made in order to apply the Ginzburg-Landau theory in this direction [3]. In the talk we present a new approach to paramagnetic-ferromagnetic transition involving a suitable order parameter related to the remnant magnetization. Starting from the skeleton-curve description, which is typical of Duhem models, and exploiting the minimum (Gibbs) free energy representation, we are able to highlight the role of the Ginzburg-Landau equation when phase transitions in materials with hysteresis are involved.

1. Atherton D. L., Jiles D. C. 1986 Theory of ferromagnetic hysteresis. J. Magn. Magn. Mater. 61, 48–60.
2. Coleman B. D., Hodgdon M. L. 1986 A constitutive relation for rate- independent hysteresis in ferromagnetically soft materials. Internat. J. Eng. Sci. 24, 897–919.
3. Fabrizio M., Giorgi C., Morro A. 2009 Phase transition in ferromagnetism. Internat. J. Engrg. Sci., 47, 821–839.
4. Visintin A., Differential Models of Hysteresis, Applied Mathematical Sciences 111, Springer, 1995.

Global attractors for Cahn-Hilliard-Navier-Stokes systems with nonlocal interactions

Maurizio Grasselli

Milano Politecnico, Milano (Italy)

A well-known diffuse interface model consists of the Navier-Stokes equations nonlinearly coupled with a convective Cahn-Hilliard equation. This system describes the evolution of an incompressible isothermal mixture of fluids and it has been investigated by many authors. Here I consider a variant of such a model where the standard Cahn-Hilliard equation is replaced by its nonlocal version. More precisely, the nonlocal interaction term in the free energy functional is no longer approximated with the gradient of the order parameter. The main goal is to present some recent results on the global longtime behavior of the (weak) solutions.

On a parabolic-hyperbolic system for contact inhibition of cell growth

Danielle Hilhorst

Paris Sud (France)

We consider a parabolic-hyperbolic system of nonlinear partial differential equations which describes a simplified model for contact inhibition of growth of two cell populations. In one space dimension it is known that global solutions exist and that they satisfy the segregation property which reflects the inhibition mechanism: if the two populations are initially segregated - in mathematical terms this is translated into disjoint spatial supports of their densities - this property remains valid for all later times. In this talk, we first use recent results on transport equations and Lagrangian flows to obtain similar results in the case of arbitrary space dimensions. Numerical experiment shows that, for certain parameter values and for a large class of initial data, the large time behavior of solutions is described by a segregated traveling wave solution with positive wave speed $\bar{c}$. We will show that for each $c > \bar{c}$ there exists a traveling wave, but for any $c > \bar{c}$, the cell densities are continuous and no longer segregated. We also show that the latter traveling waves cannot be the large time profile of the system of partial differential equations for a large class of initial functions. The structure of the traveling waves strongly resembles the one for the scalar Fisher-KPP equation, where the special role played by the traveling wave with minimal speed has been extensively studied in the literature. This is joint work with Michiel Bertsch, Hirofumi Izuhara, Masayasu Mimura and Tohru Wakasa.

Optimal control of multifrequency induction hardening

Dietmar Hömberg

WIAS, Berlin (Germany)

In my talk I will investigate a new hardening process called multi-frequency induction hardening. In contrast to standard induction heating approaches it allows for a true contour hardened pattern of gears. The mathematical model consists of a vector potential equation of Maxwells equations coupled with an energy balance and rate law to describe the growth of the high temperature phase in steel. The equations are coupled via phase dependent material parameters. In the talk I will briefly explain the model, comment on existence and uniqueness results, derive optimality conditions and conclude with some numerical simulations. Specific challenge for the numerical treatment is the resolution of the different spatial and temporal scales. To this end we employ an hp-adaptive edge element discretization of Maxwells equations and use different time scales to resolve magnetic and temperature effects, respectively. (Joint work with T. Petzold and E. Rocca)

A thermodynamically consistent hysteresis model for ferroelectricity and ferroelasticity, based on Preisach operators and hysteresis potentials

Barbara Kaltenbacher

University of Graz (Austria)

In this talk we will present a hysteresis model for piezoelectricity that has been originally proposed by Pavel Krejci (Krejci 2010) in the context of magnetostriction. It consists of an appropriate combination of Preisach type hysteresis operators with hysteresis potentials. On one hand, under certain mild sign conditions, it allows to verify the second law of thermodynamics. On the other hand, due to its very general form, it is highly flexible and therewith contains, e.g, as a special case, a model that has been devised previously (Hegewald et al 2008), which has been shown to be very well fittable to experimental results. Additionally, this model also incorporates not only ferroelectricity but also ferroelasticity.

Parabolic variational inequalities with a class of weakly time-dependent constraints and applications

Nobuyuki Kenmochi

Kyoto (Japan)

We consider parabolic variational inequalities in which a class of weakly time-dependent constraints is imposed. In case the time-dependence of constraint is smooth, from the classical thoery of evolution equations generated by time-dependent subdifferentials we see that the problem has a unique strong (i.e. differentiable in time) solution. However, in many applications to free boundary problems, we can not expect so nice time regularity for constraints as to have strong solutions. Therefore, in this talk we try to construct a solution of our variational problem in a weak sense for a class of constraints sets which are only continuous in time.

Recent news about modeling water-ice phase transitions

Pavel Krejčí

Institute of Mathematics of the Academy of Sciences of the Czech Republic, Prague (Czech Republic)

In a series of joint papers with E. Rocca and J. Sprekels, we proposed a model to explain the occurrence of large stress variations during water freezing and ice melting. This is done by considering different specific volumes of water and ice, as well as the mechanical reaction of the container. The balance laws of energy, entropy, and momentum are transformed into a system of one PDE, one variational inequality, and one integrodifferential equation with or without hysteresis, depending on whether or not the container exhibits plastic memory effects. Recently, we have removed some of the simplifying modeling assumptions and constructed a global solution also in the case that different values in water and ice of further physical parameters (specific heat capacity, speed of sound, heat conductivity) are taken into account. This produces additional higher order nonlinearities in the balance equations, which have to be controlled by means of a refined version of the Moser-Alikakos scheme.

A phase-field approximation to the Willmore flow with constraints

Philippe Laurençot

Toulouse (France)

A phase-field approximation to the Willmore flow with either a volume constraint or area and volume constraints is studied and shown to be well-posed. The existence of solutions relies on the gradient flow structure of the problem (joint works with Pierluigi Colli).

Some equations with logarithmic nonlinear terms

Alain Miranville

Poitiers (France)

Our aim in this talk is to discuss the well-posedness and the asymptotic behavior of several equations with logarithmic nonlinear terms. Such equations arise, e.g., in phase separation and transition.

A quasistatic evolution model for perfectly plastic plates derived by Gamma-convergence

Maria Giovanna Mora

Pavia (Italy)

The subject of this talk is the rigorous derivation of a quasistatic evolution model for a linearly elastic - perfectly plastic thin plate. As the thickness of the plate tends to zero, we show via Gamma-convergence techniques that solutions to the three-dimensional quasistatic evolution problem of Prandtl-Reuss elasto-plasticity converge to a quasistatic evolution of a suitable reduced model. In this limiting model the admissible displacements are of Kirchhoff-Love type and the stretching and bending components of the stress are coupled through a plastic flow rule. This is based on a joint work with Elisa Davoli (SISSA).

PDEs for polymer phase separation

Irena Pawlow

Warsaw (Poland)

The goal of the talk is twofold. First we recall the Flory-Huggins-de Gennes free energy and the associated Cahn-Hilliard-de Gennes model for isothermal polymer phase separation. Secondly, we present a thermodynamically consistent nonisothermal polymer phase separation model. This is a system of the Cahn-Hilliard-de Gennes equation with temperature dependent coefficients coupled with a modified energy equation. The derivation of the nonisothermal model is based on the exploitation of the second law in the form of the Müller-Liu entropy inequality. Basic thermodynamic properties of this model, including the Lyapunov property and energy estimates, are presented. Moreover, the connections to the Penrose-Fife phase-field model with conserved order parameter are discussed.

Coupling hyperbolic heat conduction with mechanical vibrations and laser-induced defect dynamics in crystalline media: a challenge for the modeler and for the analyst

Paolo Podio-Guidugli

Roma (Italy)

Laser heating of crystalline materials induces creation/annihilation of lattice defects; defect diffusion is strongly coupled with thermal and mechanical dynamical phenomena, under form of dissipative waves. In my talk, I plan to discuss a mathematical model for this complex phenomenology.

As to defect dynamics, the mathematical format I adopt (and adapt) is the one I proposed to model phase transition in crystalline materials [1], a format that has been found mathematically robust and sound [2,3] (for brevity, I here quote the first and last - so far - of a series of papers written in collaboration with P. Colli, G. Gilardi, and J. Sprekels). As to thermal waves, I exploit the 'hyperbolicizing' effect on the heat equation of splitting entropy into a standard and a dissipative part [4].

[1] P-G, Models of phase segregation and diffusion of atomic species on a lattice. Ric. Mat. 55 (1) (2006)
[2] C, G, P-G, and S, Existence and uniqueness of a global-in-time solution to a phase segregation problem of the Allen-Cahn type. MMMAS 20 (4) (2010)
[3] C, G, P-G, and S, Global existence and uniqueness for a singular/degenerate Cahn-Hilliard system with viscosity. arXiv:1205.4979v1 (2012)
[4] P-G, Dissipation entropy makes the heat equation hyperbolic. AAPP (Cl. Sci. Fis. Mat. Nat.) 90 (1) (2012)

Shape memory effects in thermal retraction of Polyethylene

Francesco Scavello

Versalis-ENI, Mantova (Italy)

Semycrystalline polymeric materials can show thermal retraction on heating when the molecular structure has been oriented as a consequence of the application of a mechanical strain. Thermal retraction can be interpreted as a shape memory behavior. In this work we describe the one-dimensional thermomechanical behavior of the oriented polyethylene using the "phase transition" approach. As the internal variable, characterizing the state of the material, we take into consideration the amount of the oriented phase. On this basis, we can successfully describe the stress-strain hysteresis, characteristic for this class of materials. We also define a set of parameters capable of describing the mechanical behavior of the material at different temperatures. Furthermore, we tested the model both in terms of "free thermal retraction" (i.e. the shape recovery of the oriented material on heating) and "shrinkage force" (i.e. the reaction force exerted when the shape recovery is inhibited), finding a good agreement with experiments. This is a joint work with E. Bonetti, L. Castellani, M. Pachera.

On some Cahn-Hilliard models with nonlinear diffusion

Giulio Schimperna

Pavia (Italy)

In this talk we will focus on a class of Cahn-Hilliard equations characterized by nonlinear diffusion dynamics. These models, which arise in the physics of polymers and of water-oil-surfactant mixtures, have an independent mathematical interest as they can be viewed as gradient flow problems for the free energy functional

$$E(u) = \int_\Omega \left(\frac{a(u)}{2} |\nabla u|^2 + F(u) \right),$$

where $F(u)$ is the standard logarithmic potential and $a(u)$ is a nonlinear function giving rise to the nonlinear diffusion effect in the equation. We will first consider the case when a is smooth, bounded and strictly positive. Then, we will consider the singular case corresponding to $a(u) \sim (1-u^2)^{-1}$. In both cases we will prove existence of a weak solution. Moreover, we will discuss about uniqueness and parabolic regularization properties, which hold in special cases. The results discussed in this talk have been obtained in collaboration with Irena Pawlow (Military University of Technology and Polish Academy of Sciences, Warsaw).

A time discretization for a nonstandard viscous Cahn-Hilliard system

Jürgen Sprekels

WIAS, Berlin (Germany)

In this talk, we make the first step towards the derivation of a numerical method for the approximate solution of a nonstandard viscous Cahn-Hilliard system describing phase separation on an atomic lattice. This system, which has been studied recently in a series of papers by P. Colli, G. Gilardi, P. Podio-Guidugli and the lecturer, is a strongly nonlinearly coupled system of two PDEs for an order parameter and the chemical potential. In this talk, we propose a time discretization of this system. We prove unique existence of the discrete solutions and their convergence to the continuous solution, and we present an error analysis.

Finite plasticity Γ-converges to linearized plasticity

Ulisse Stefanelli

Pavia (Italy)

I shall comment on the possibility of passing to limits in evolution rate-independent systems. In particular, I will outline a general *evolutive* Γ-*convergence* theory for energetic solutions.

The abstract theory will then be applied in the context of finite plasticity providing a rigorous justification of the classical linearization approach.

This is joint work with A. Mielke and T. Roubíček.

(Non)local phase transitions and minimal perimeter interfaces

Enrico Valdinoci

Milano (Italy)

We would like to present some problems related to local and nonlocal Allen-Cahn-type equations, with respect to the symmetry and rigidity properties of the solutions and of the interfaces. Some open problems will be also discussed.

CONTRIBUTED TALKS

A temperature-dependent model for adhesive contact with friction

Giovanna Bonfanti

Brescia (Italy)

The present talk concerns the investigation of a contact problem between a viscoelastic body and a rigid support, when the effects due to adhesion, friction and the evolution of temperature are taken into account. In particular, we allow for the temperature to influence the intensity of the adhesion, so that we consider thermal effects both in the bulk domain and on the contact surface. Moreover, we include the contribution due to friction in the heat source on the contact surface and we allow for the friction coefficient to depend on the the temperatures of the body and of the adhesive material. We describe the adhesion phenomenon in terms of a damage surface parameter according to Frémond's theory, we model unilateral contact by the Signorini conditions, and friction by a nonlocal Coulomb law. Moreover, entropy balance laws govern the evolution of the temperatures. The resulting PDE system is highly nonlinear: the main analytical difficulties are related to the presence of multivalued operators rendering physical constraints on the variables, as well as to the contact and the friction conditions, to the singular character of the temperature equations, and to the nonlinear coupling between the equations themselves. We prove global-in-time existence results. The presented results have been obtained in collaboration with Elena Bonetti and Riccarda Rossi.

Existence for the steady problem of a mixture of two power-law fluids

Hermenegildo Borges de Oliveira

Lisboa (Potugal)

The steady problem resulting from a mixture of two distinct fluids of power-law type is analyzed in this talk. Mathematically, the problem results from the superposition of two power laws, one for a constant power-law index with other for a variable one. For the associated boundary-value problem, we prove the existence of very weak solutions, provided the variable power-law index is bounded from above by the constant one.

Long-time behaviour of a simplified Ericksen-Leslie non-autonomous system for nematic liquid crystal flows

Stefano Bosia

Milano Politecnico (Italy)

We analyze a simplified Ericksen-Leslie model for nematic liquid crystal flows first introduced by Lin and Liu, with non-autonomous forcing bulk term and boundary conditions on the order parameter field. We obtain existence of weak solutions in the two- and three-dimensional cases. We prove uniqueness,

continuous dependence on initial conditions, forcing and boundary terms and also existence of strong solutions in the 2D case. Focusing on the 2D case, we then study the long term behaviour of solutions by obtaining existence of global attractors for normal forcing terms. Finally, we prove the existence of exponential attractors for quasi-periodic forcing terms in the 2D model.

A phase field model: well-posedness, regularity, and asymptotic behaviour with respect to some special parameters

Giacomo Canevari

Paris VI (France)

This talk reports about a joint work with P. Colli. A diffusion model, of phase-field type, is considered. As a special feature of this model, the heat flux involves two contributions: a term shaped by the Fourier's law, and another one taking into account possible memory effects. Furthermore, the hypotheses on the bulk potential are general enough to allow non-smooth, multivalued graphs to be considered. Firstly, the well-posedness of the problem is stated. Then, several regularity results, implying that the solution is strong and with bounded components, are provided. Finally, we study the limit as the two characteristic parameters of the model, governing the size of the terms in the heat flux, tend to zero, separately; in both cases, convergence results are stated, as well as estimates on the convergence speed.

Free energy functionals and solutions of viscoelasticity problems

Sandra Carillo

Roma La Sapienza (Italy)

Some linear integro-differential problems characterized by a nonlocal kernel are considered. Specifically some new results, obtained with V. Valente and G. Vergara Caffarelli [1], on a singular viscoelasticity problem are presented. Notably, as already pointed out in previous works [2-3] an important role is played by the choice of the free energy functional. Hence, the problem of the choice of the free energy, studied in various different physical models, in jont work with G. Amendola and A. Manes [4,5], is also considered.

[1] Carillo S., Valente V. and Vergara Caffarelli G. A result of existence and uniqueness for an integro-differential system in magneto-viscoelasticity, Applicable Analisys: An International Journal, 1791–1802, (90) n.ro 12, ISSN: 0003-6811, doi: 10.1080/00036811003735832. 2011;

[2] Carillo S., Valente V. and Vergara Caffarelli G. An existence theorem for the magneto-viscoelastic problem Discrete and Continuous Dynamical Systems Series S, 435–447, (5) n.ro 3., ISSN: 1937-1632, doi:10.3934/dcdss.2012.5.435. 2012;

[3] Carillo S., Valente V. and Vergara Caffarelli G. A singular viscoelastic problem: existence and uniqueness results. preprint. 2012;

[4] Amendola G., Carillo S. and Manes A. Classical free energies of a heat conductor with memory and the minimum free energy for its discrete spectrum model, Bollettino U. M.l., sect. B, 421 – 446 (3) n.ro 3, ISSN: 1972-6724. 2010

[5] Amendola, G., Carillo S. and Manes, A., Free energies for viscoelastic fluids. Differential problems and asymptotic behaviour. preprint 2012.

On a 3D isothermal model for nematic liquid crystals accounting for stretching terms

Cecilia Cavaterra

Milano (Italy)

We investigate the well-posedness of a PDE system for the velocity field $\mathbf{u}$ and the director field $\mathbf{d}$, describing the evolution of a nematic liquid crystal flow under kinematic transports for molecules of different shapes. The evolution of u is ruled by the Navier-Stokes incompressible system with a stress tensor exhibiting a special coupling between the transport and the induced terms. The dynamic of d is described by a variation of a parabolic Ginzburg-Landau equation with a suitable penalization of the physical constraint $|\mathbf{d}| = 1$. The main aim is to overcome the lack of a maximum principle for the director equation and prove the existence of global in time weak solutions under physically meaningful boundary conditions on $\mathbf{u}$ and $\mathbf{d}$.

Attractors for processes on time-dependent spaces and applications to the wave equations

Monica Conti

Milano Politecnico (Italy)

We discuss a recent theory of pullback attractors for processes $U(t,\tau) : X_\tau \to X_t$ acting on time-dependent normed spaces. The new scheme is applied to study the longterm behavior of wave equations with time-dependent speed of propagation. This is a joint work with Vittorino Pata and Roger Temam.

Asymptotics of fractional perimeter functionals

Serena Dipierro

Sissa, Trieste (Italy)

We deal with the asymptotic behavior of the s-perimeter of a set E inside a domain Ω as $s \searrow 0$. We discuss necessary and sufficient conditions for the existence of such limit, by also providing an explicit formulation in terms of the Lebesgue measure of E and Ω. Moreover, we construct examples of sets for which the limit does not exist.

Some problems associated with the second order optimal shape of a crystallisation interface

Pierre-Éthienne Druet

WIAS Berlin (Germany)

We discuss the optimal control of second order quantities (main curvatures) for a free surface subject to the Stefan problem with surface tension.

Three different approaches to the Souza-Auricchio model for shape memory alloys

Michela Eleuteri

Milano (Italy)

We describe three different approaches starting from the Souza-Auricchio model for shape memory alloys. First we analyze a rate-independent model including permanent inelastic effects; second we study the thermal control of the Souza-Auricchio model. Finally we propose a different regularization, based on the Preisach model for hysteresis, to have the thermodynamic consistency of the Souza-Auricchio model and we prove well-posedness of the full system of dynamical balance equations.

Asymptotic analysis of some isothermal models for nematic liquid crystal flows

Sergio Frigeri

Milano (Italy)

We shall first review some known asymptotic behavior results of solutions for some models for the flow of nematic liquid crystals. In particular, we shall focus on a model, introduced by Sun and Liu, which represents a correction to the Lin-Liu simplified version of the original Ericksen-Leslie model and which is physically relevant since it accounts for stretching and rotation effects of molecules. For the Sun-Liu model we shall present some recent results concerning the asymptotic behavior in 3D (existence of trajectory attractors) for weak solutions under different boundary conditions for the director field and under different assumptions for the double-well potential. Joint work with Elisabetta Rocca.

On a variational inequality of Bingham and Navier-Stokes type in three dimension

Takeshi Fukao

Kyoto (Japan)

In this talk, we discuss the well-posedness of the variational inequality for a fluid dynamics of Bingham and Navier-Stokes type in three dimension. This kind of problem was treated by J. Naumann (1980), Y. Kato (1993) for the Bingham fluid, based on the result by G. Duvaut and J. L. Lions (1976). Moreover by J. F. Rodrigues (2000) and the references therein, the system coupled with the Stefan problem was treated. In three dimension, some mathematical problem still remains to be done. By modifying the problem to the variational inequality with the velocity constraint, we discuss the well-posedness related to this problem.

A variational approach to a Cahn-Hilliard model in a domain with non-permeable walls

Stefania Gatti

Modena (Italy)

This talk is devoted to the well-posedness and the long time behavior of a Cahn-Hilliard model with a singular bulk potential and suitable dynamic boundary conditions. We assume here that the system is

confined in a vessel with non-permeable walls and that the total mass, in the bulk and on the boundary, is conserved. As a result, the well-posedness in the sense of distributions may not hold and new notions of solutions are required. The same problem has been analyzed by G. Ruiz-Goldstein, A. Miranville and G. Schimperna, relying on duality techniques, under weak assumptions on the nonlinearities. However, the regularity of solutions and the study of the asymptotic behavior of the system required growth restrictions on the bulk nonlinearity which exclude the thermodynamically relevant logarithmic potentials. The aim of the talk is to show a wider analysis by introducing a variational formulation of the problem, based on a proper variational inequality. These results have been obtained in a joint work with L. Cherfils and A. Miranville.

Representation of Hysteresis operators for vector-valued inputs by string functions

Olaf Klein

WIAS Berlin (Germany)

In [BrokSpre-1996], Brokate and Sprekels have derived an representation formulae for hysteresis operators acting on scalar-valued continuous input functions being piecewise monotone. In [K.-2012a, K.-2012b], hysteresis operators dealing with inputs in a general topological vector space V are investigated, and a similar representation formulae is derived.

To define an appropriate generalization of monotonicity for scalar function for functions with values in a vector space V, the composition of a **monot**one with an **affine** function leads to a **monotaffine** function as defined in [K.-2012a,K.-2012b].

For every piecewise monotaffine functions u the *standard monotaffinicity partition of* $[0,T]$ *for* u is the uniquely defined decomposition $0 = t_0 < t_1 < \cdots < t_N = T$ of $[0,T]$ such that for $1 \le i \le N$ t_i is the maximal number in $]t_{i-1}, T]$ such that u is monotaffine on $[t_{i-1}, t_i]$.

As generalization of definition for the scalar case in [BrokSpre-1996], $(v_0, \ldots, v_N) \in V^N$ is denoted as *convexity triple free string*, if for all $1 \le i < N$ and it holds that v_i is not a convex combination of v_{i-1} and v_{i+1}. Let $S_F(V)$ be the set of all convexity triple free strings of elements of V.

In [K.-2012b] it is is shown, that for every hysteresis operator $\mathcal{H}$ acting on continuous, piecewise monotaffine inputs exists a unique function $G : S_F(V) \to Y$ such that for all $u \in C_{\text{p.m.a.}}([0,T]; V)$ and the corresponding standard monotaffinicity decomposition $0 = t_0 < t_1 < \cdots < t_N = T$ it holds that

$$\mathcal{H}[u](t) = G\left(u(t_0), u(t)\right), \qquad \forall\, t \in [t_0, t_1], \tag{0.1}$$

$$\mathcal{H}[u](t) = G\left(u(t_0), \ldots, u(t_{i-1}), u(t)\right), \qquad \forall\, t \in]t_{i-1}, t_i],\ 2 \le i \le N. \tag{0.2}$$

Moreover, it is shown that for every function $G : S_F(V) \to Y$ there exists a corresponding hysteresis operator acting on continuous, piecewise monotaffine inputs.

BrokSpre-1996 M. Brokate, J. Sprekels, *Hysteresis and Phase Transitions*, Springer-Verlag, New York, 1996.

K.-2012a O. Klein, *Representation of hysteresis operators for vector-valued inputs by functions on strings*, Physica B, 407 (2012), 1399–1400,

`http://dx.doi.org/10.1016/j.physb.2011.10.015`

K.-2012b O. Klein, *Representation of hysteresis operators for vector-valued continuous monotaffine input functions by functions on strings*, WIAS Preprint 1698

Non-isothermal cyclic fatigue in an oscillating elastoplastic material with phase transition

Jana Kopfova

Opava (Czech Republic)

We study a temperature dependent model of fatigue accumulation in an oscillating elastoplastic material. Our model is based on the hypotheses that there exists a proportionality between fatigue and dissipated energy and is inspired by the idea of melting the material in the place of bending in order to repair it, so the fatigue rate can also decrease in time. This is a joint work with Petr Sander and Pavel Krejci

---

Bio-materials and chemotaxis

Gabriela Marinoschi

Iasi (Romania)

This paper investigates the well-posedness of a reaction-diffusion system of chemotaxis type, with a nonlinear diffusion coefficient and a dynamics (growth-death) of the cell population b.

Decay of solutions for a thermoelastic mixture

Mariagrazia Naso

Brescia (Italy)

Our aim in this talk is to discuss a PDE system modeling thermomechanical deformations for mixtures of thermoelastic solids. In particular we investigate the asymptotic behavior of the related solutions. First, we find necessary and sufficient conditions to guarantee that the solutions decay exponentially. Subsequently, when the decay is not of exponential type, we prove that the solutions decay polynomially and we find the optimal polynomial decay rate.

On the omega-limit set for a nonlocal evolution problem

Thanh Nam Nguyen

Paris Sud (France)

We consider an initial value problem for a nonlocal differential equation with a bistable nonlinearity and discuss the large time behavior of the solution orbits for a large class of initial functions. More precisely, we show that if the initial function is strictly piecewise monotone, its omega-limit set contains exactly one element, which has the form of a step function, and takes at most two values. This is joint work with Danielle Hilhorst and Hiroshi Matano.

An isogeometric collocation method for Cahn-Hilliard phase separation

Alessandro Reali

Pavia (Italy)

This is a joint work with Hector Gomez (University of A Coruna) and Giancarlo Sangalli (University of Pavia). Isogeometric Analysis (IGA) is a recent idea introduced by Hughes et al. to bridge the gap between Computational Mechanics and CAD. The key feature of IGA is to extend the finite element method representing geometry by functions (e.g., NURBS) which are used by CAD systems, and then invoking the isoparametric concept to define field variables. Thus, the computational domain exactly reproduces the NURBS description of the physical domain. Numerical testing in different situations has shown that IGA holds great promises, with a substantial increase in the accuracy-to-computational-effort ratio with respect to standard finite elements. In the framework of NURBS-based IGA techniques, collocation methods have been recently proposed constituting an interesting high-order low-cost alternative to standard Galerkin approaches and providing good results in the fields of elastostatics and elastodynamics. In this talk, an isogeometric collocation method is developed and tested for the simulation of the phase separation model described by the Cahn-Hillard equation. The linearly stabilized Eyre's method is adopted to perform time integration. Several numerical experiments are carried out in order to show the behavior and the advantages of the proposed approach.

Analysis of a degenerating PDE system for phase transitions and damage

Riccarda Rossi

Brescia (Italy)

Based on Frémond's approach, we propose a PDE system for the modeling of phase transition and damage in thermoviscoelastic materials. The resulting evolution equations for the absolute temperature, the displacement, and the phase/damage parameter, are strongly nonlinearly coupled. Moreover, the momentum equation contains elliptic operators, which degenerate at the pure phases (corresponding to the threshold values for the phase change/damage parameter), making the whole system degenerate.

We consider a a suitable notion of weak solution for the analysis of this problem. We prove an existence result by passing to the limit in an approximating problem, where the elliptic degeneracy of the displacement equation is ruled out, via suitable variational techniques. This is joint work with Elisabetta Rocca.

A sixth order Cahn-Hilliard type equation

Piotr Rybka

Warsaw (Poland)

We study a sixth order convective Cahn-Hilliard type equation type that describes the faceting of a growing surface is considered with periodic boundary conditions. We deal with the problem in one and two dimensions. We establish the existence and uniqueness of weak solutions. We also address the long time behavior in the on-dimensional case

---

Some remarks on an ultra fast diffusion equation

Antonio Segatti

Pavia (Italy)

In this talk I will report on some results obtained with G. Schimperna and S. Zelik about smoothing effects for solutions to ultra fast diffusion equations in domanis with dynamic boundary conditions. Possible applications to phase transitions models of Penrose Fife will be presented.

Homogenization of laminate single-negative metamaterials

Helia Serrano

Castilla-La Mancha (Spain)

We study the asymptotic behaviour of the magnetic induction of an artificial composite material formed by alternate layers of two metamaterials, with equal negative magnetic permeability and different positive electric permittivity, through the homogenization of a second-order initial-boundary value problem coming from the non- stationary Maxwell equations.

A rate-independent model for brittle delamination in thermoviscoelasticity

Marita Thomas

WIAS Berlin (Germany)

We address a model for a brittle delamination process along a prescribed interface in a thermoviscoelastic body. In the spirit of continuum damage mechanics, delamination is modeled by means of an internal variable, governed by a rate-independent flow rule. The lattter is coupled in a highly nonlinear way with the heat equation and the momentum balance for the displacements, featuring frictionless Signorini conditions on the interface where delamination occurs. We present existence and approximation results for a suitable weak formulation of the delamination system, obtained in collaboration with Riccarda Rossi (Brescia).

Compressible Phase Change Flows and the Existence of Transition Profiles

Gabriele Witterstein

WIAS Berlin (Germany)

We consider diffusive interface models describing flows which undergo a phase change. The equations consist of the compressible Navier-Stokes system coupled with an Allen-Cahn equation (phase field equation), and are based on an energetic variational formulation. We concentrate on a model where a jump in the mass density on the free boundary in the sharp interface limit arise. Here, the central point is to show the existence of the transition profiles in the interface region connecting the bulk regions.
