Vector calculus MA2VC 2013–14 — Assignment 2

Handed out: Friday 1st November. Due: **Friday 15th November, 12 noon**.
Late assignments will not be accepted.
You can use the vector identities in the lecture notes.
Total marks: 25. (5% of the total marks for the module.)

(1) (6 marks) Consider the curve

$$\vec{\mathbf{a}}(t) = e^t\hat{\boldsymbol{\imath}} + t\hat{\boldsymbol{\jmath}} + e^{2t}\cos t\,\hat{\boldsymbol{k}}, \qquad t \in \mathbb{R},$$

and the scalar field $f = |\vec{\mathbf{r}}|$.

- Compute the total derivative $\frac{d\vec{\mathbf{a}}}{dt}$ of the curve.
- Compute the total derivative $\frac{d(f(\vec{\mathbf{a}}))}{dt}$ of the evaluation of the field on the curve; write it as a function of t only (i.e. not containing x, y, z).

(2) (5 marks) Consider the curve

$$\vec{\mathbf{b}}(t) = 5\cos t\,\hat{\boldsymbol{\imath}} + \big(3\cos t - 4\sqrt{2}\sin t\big)\hat{\boldsymbol{\jmath}} + \big(4\cos t + 3\sqrt{2}\sin t\big)\hat{\boldsymbol{k}}.$$

Compute the length of the path of $\vec{\mathbf{b}}$ corresponding to the interval $0 \le t \le \pi$.

(3) (8 marks) Consider the two curves

$$\vec{\mathbf{c}}(t) = \frac{2}{\pi}t\,\hat{\boldsymbol{\imath}} + \cos t\,\hat{\boldsymbol{\jmath}}, \qquad \text{for } -\frac{\pi}{2} \le t \le \frac{\pi}{2},$$

$$\vec{\mathbf{d}}(\tau) = \tau\,\hat{\boldsymbol{\imath}} + (1-\tau^2)\hat{\boldsymbol{\jmath}}, \qquad \text{for } -1 \le \tau \le 1,$$

and the vector field $\vec{\mathbf{G}} = y\hat{\boldsymbol{\imath}}$. Compute the integrals of $\vec{\mathbf{G}}$ along the two paths described by the two curves and use the result obtained to prove that $\vec{\mathbf{G}}$ is not conservative. (A sketch of the two paths may help you.)

How can you prove (easily) that $\vec{\mathbf{G}}$ is not conservative without computing any integral?

(4) (6 marks) Show that the vector field $\vec{\mathbf{H}} = x\hat{\boldsymbol{\imath}} + y^2\hat{\boldsymbol{\jmath}} + z^3\hat{\boldsymbol{k}}$ is irrotational and compute a scalar potential. Use the scalar potential obtained to compute the line integral of $\vec{\mathbf{H}}$ along one of the paths described in Exercise 3.