MA2VC, Vector Calculus, Assignment 4

due: 12pm, 14 Dec 2012 (late assignments will not be accepted, and marks will be deducted for poor presentation)

Consider the vector field

$$\mathbf{F}(\mathbf{r}) = xy\hat{\mathbf{j}}$$

1a) (7 marks) Evaluate the surface integral

$$\oint_{\partial R} \mathbf{F} \cdot \hat{\mathbf{n}} dS$$

where $\hat{\mathbf{n}}$ is the outward-pointing normal on the surface, ∂R, of the tetrahedron, R, defined by $x \geq 0$, $y \geq 0$, $z \geq 0$, and $3x + 2y + z \leq 6$.

1b) (1 mark) Explain why the surface integral has the same value as the volume integral

$$\int_R x dV$$

calculated in assignment 3.

2a) (6 marks) Evaluate the surface integral

$$\oint_{\partial D} \mathbf{F} \cdot \hat{\mathbf{n}} dS$$

where $\hat{\mathbf{n}}$ is the outward-pointing normal on the surface, ∂D, of the hemisphere, D, defined by $0 \leq x \leq \sqrt{1 - y^2 - z^2}$ where $y^2 + z^2 \leq 1$. Hint, project the spherical surface into the y-z plane and use polar coordinates.

2b) (6 marks) Evaluate the volume integral

$$\int_D \nabla \cdot \mathbf{F} dV$$

Hint, use spherical-polar coordinates.