MA2VC, Vector Calculus, Assignment 4

due: 12pm on the 16th of Dec 2011 (late assignments will not be accepted)

1) (20 marks) Demonstrate that the divergence theorem

$$\int_D \nabla \cdot \mathbf{F} \, dV = \oint_{\partial D} \mathbf{F} \cdot \hat{\mathbf{n}} \, dS$$

holds for the vector field, $\mathbf{F} = 2x\hat{\mathbf{i}} - x^2\hat{\mathbf{j}} + (z - 2x + 2y)\hat{\mathbf{k}}$, where D is the domain defined by $x \geq 0$, $y \geq 0$, $z \geq 0$ and $2x + 2y + z \leq 6$. That is confirm that the integral of the divergence over D equals the surface integral of $\mathbf{F} \cdot \hat{\mathbf{n}}$ over the 4 sides of D.