MA2VC, Vector Calculus, Assignment 1

due: 12pm on the 4th of Nov 2011 (late assignments will not be accepted)

1) (7 marks) Prove the identity:

$$\nabla \cdot (\mathbf{F} \times \mathbf{G}) = (\nabla \times \mathbf{F}) \cdot \mathbf{G} - \mathbf{F} \cdot (\nabla \times \mathbf{G})$$

2) (7 marks) Demonstrate that the above identity is satisfied for the vector fields:

$$\begin{aligned} \mathbf{F} &= yz\hat{\mathbf{i}} + xz\hat{\mathbf{j}} \\ \mathbf{G} &= x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}} \end{aligned}$$

3a) (2 marks) Evaluate $\mathbf{F} = \nabla \times \mathbf{A}$, where $\mathbf{A} = xz\hat{\mathbf{i}} - yz\hat{\mathbf{j}}$.
3b) (2 marks) Show that it is both an irrotational vector field (*i.e.*, $\nabla \times \mathbf{F} = 0$) as well as a solenoidal vector field (*i.e.*, $\nabla \cdot \mathbf{F} = 0$).
3c) (2 marks) Determine its scalar potential ϕ (*i.e.*, $\mathbf{F} = \nabla\phi$).